Source task and conventions

Classify the source's causal M-sample moving average, $Tx[n] = M^{-1} \sum_{k=0}^{M-1} x[n-k]$, for integer $M \geq 1$.

Conventional solution and proof

For scalars a, b, distributing the finite sum proves $T(ax + bv) = aTx + bTv$. If $|x[n]| \leq B$, then $|Tx[n]| \leq M^{-1} \sum_{k=0}^{M-1} B = B$, proving BIBO stability. Only indices $n - k \leq n$ occur, proving causality. For a shift $S_r x[n] = x[n-r]$, substitution gives $T(S_r x)[n] = M^{-1} \sum_k x[n-r-k] = (S_r Tx)[n]$, proving time invariance. The induced ℓ^∞ norm equals one: the upper bound is attained by $x[n] \equiv 1$.

Structural or analytic derivation

Its impulse response is $h[k] = 1/M$ on $0 \leq k < M$ and zero elsewhere. The finite support and sum $\|h\|_1 = 1$ encode causality and the exact gain. A rolling sum satisfies

$$y[n] = y[n-1] + \frac{x[n] - x[n-M]}{}.$$