Source task and conventions

Classify the source's discrete differentiator $Dx[n] = (x[n] - x[n-1])/T$, where $T > 0$.

Conventional solution and proof

Distributing the two terms proves linearity. The output uses the present and previous input, so it is causal. Replacing $x[n]$ by $x[n-r]$ shifts the output by the same r, proving time invariance. For bounded input $|x[n]| \le B$, $|Dx[n]| \le 2B/T$, so the discrete operator is BIBO stable. The alternating input $x[n] = B(-1)^n$ attains the bound; the exact induced norm is $2/T$.

Structural or analytic derivation

The impulse response is $T^{-1}(\delta[n] - \delta[n-1])$, and

$$H(e^{i\omega}) = T^{-1}(1 - e^{-i\omega}) = \frac{2i}{T} e^{-i\omega/2} \sin(\omega/2).$$

Its maximum magnitude is $2/T$. Taylor's integral identity gives $[x(t) - x(t-T)]/T = x'(t) - T^{-1} \int_{t-T}^{t} (s - t + T) x''(s)\, ds$; hence the derivative error is at most $T \sup |x''|/2$. This distinguishes exact discrete stability from the approximation to continuous differentiation.

Comparative cost and preferred method

One subtraction and one scale already realize the exact operator. The frequency representation gives no cheaper sample implementation; it provides the sharp gain and an accuracy condition. As $T \downarrow 0$, the norm grows, so stability for each fixed sample period does not imply a uniform noise bound.