Source task and conventions

Express the three requested length-N DFTs in terms of $X[k]$, using the negative-exponent forward convention.

Conventional solution and proof

Let $\bar{k} = (-k) \bmod N$. Direct conjugation gives

$$\mathcal{D}\{x^*\}[k] = X^*[\bar{k}].$$

Since $\Re x = (x + x^*)/2$ and $i\Im x = (x - x^*)/2$, linearity gives

$$\boxed{\mathcal{D}\{\Re x\}[k] = \tfrac{1}{2}(X[k] + X^*[\bar{k}]), \quad \mathcal{D}\{i\Im x\}[k] = \tfrac{1}{2}(X[k] - X^*[\bar{k}]).}$$

The identity $e^{2\pi ikn/N} = e^{-2\pi i\bar{k}n/N}$ proves that the reversal is circular, including $k = 0$.

Structural or analytic derivation

These are projections induced by the conjugation-reversal involution on coefficient space. Applying either projection twice leaves it unchanged, as follows by replacing k with $\bar{k}$ and conjugating. This gives a second algebraic consistency check.

Comparative cost and preferred method

All outputs require $O(N)$ additions and conjugations once X is known, compared with $O(N \log N)$ for independent FFTs. Reading or writing N outputs supplies the matching $\Omega(N)$ output lower bound.