Source task and conventions

Starting from $y[n] = ca^n u[n]$, $a = e^{-\alpha}$, transform $ny[n]$, $\beta^n y[n]$, $y[-n]$ and $y[n - n_0]$.

Conventional solution and proof

For $a \neq 0$ and finite c, $Y(z) = c/(1 - az^{-1})$, $|z| > |a|$. Termwise differentiation on compact subannuli gives

$$\mathcal{Z}\{ny[n]\} = -zY'(z) = \frac{caz^{-1}}{(1 - az^{-1})^2}, \quad |z| > |a|.$$

Scaling the coefficient base gives $\mathcal{Z}\{\beta^n y[n]\} = c/(1 - a\beta z^{-1})$, $|z| > |a\beta|$; if $\beta = 0$, the sequence is $c\delta[n]$. Reversal gives $Y(z^{-1}) = c/(1 - az)$ with $|z| < 1/|a|$. For integer n_0, substitution $m = n - n_0$ gives $z^{-n_0}Y(z)$, with the same finite nonzero radial boundary; zero/infinity inclusion is determined by the resulting finite shift. The identically zero case $c = 0$ has no genuine poles.

Structural or analytic derivation

The four operations are the Euler differential operator, a dilation of the argument, inversion on the Riemann sphere and a Laurent monomial. Their convergence domains follow from the transformed series, not solely from the rational denominator.

Comparative cost and preferred method

Each symbolic transformation has fixed cost once Y is known. Re-summing an infinite sequence is unnecessary. The standard table operations already realize this reduction; the analytic contribution is explicit domain tracking and removable-degeneracy handling.