Source task and conventions

Analyze the rational filter in (i) and the displayed input-dependent recurrence in (ii), retaining its literal multiplication.

Conventional solution and proof

For (i), clearing negative powers gives

$$H(z) = \frac{z(z-1)}{(z^2 - z + 1)(z + 4/5)}.$$

Zeros are $0, 1$ and poles are $e^{\pm i\pi/3}, -4/5$. For the causal interpretation the unit-circle poles do not cancel, so the impulse response is not absolutely summable and the filter is not BIBO stable. More generally no ordinary stable bilateral realization can have a noncancelled pole on its convergence circle.

Part (ii) literally reads $3y[n] = 3.7y[n-1] - 0.7y[n-2]x[n-1]$. Its state transition depends on the input, so it is not an LTI input-output recurrence and has no single transfer function whose poles can be plotted. With zero initial state it produces the identically zero output for every input. With a nonzero state and bounded input $x \equiv 0$, it includes the growing mode $(37/30)^n$, so internal stability fails. An additive forcing term or an intended plus sign is not supplied by the source.