Source task and conventions

Realize $H(z) = (1 - \alpha z^{-1} - \beta z^{-2})^3$ directly and as three second-order sections.

Conventional solution and proof

Put $q = z^{-1}$. Polynomial multiplication gives

$$H = 1 - 3\alpha q + (3\alpha^2 - 3\beta)q^2 + (6\alpha\beta - \alpha^3)q^3 + (3\beta^2 - 3\alpha^2\beta)q^4 - 3\alpha\beta^2 q^5 - \beta^3 q^6.$$

The direct realization is a six-delay tapped line with these seven coefficients, $y[n] = \sum_{k=0}^{6} b_k x[n-k]$. The cascade is $v_0 = x$ and, for $j = 1, 2, 3$,

$$v_j[n] = v_{j-1}[n] - \alpha v_{j-1}[n-1] - \beta v_{j-1}[n-2], \qquad y = v_3.$$

Each stage has two delays. Taking Z transforms with zero states gives $V_j = (1 - \alpha q - \beta q^2)V_{j-1}$; multiplication proves exact realization equality.

Structural or analytic derivation

The cascade retains the repeated polynomial factor and avoids forming powers of coefficients at setup. Both realizations have order at most six; special parameter cancellations can lower the effective order. The diagram gives each second-order signal equation and its delay states.

Comparative cost and preferred method