Source task and conventions

Give direct-form II, first/second-order cascade, and two-biquad cascade realizations of the stated third-order transfer function.

Conventional solution and proof

With $q = z^{-1}$ the expanded numerator and denominator are

$$B(q) = 1 + \tfrac{4}{5}q + \tfrac{17}{100}q^2 + \tfrac{1}{100}q^3, \quad A(q) = 1 + \tfrac{17}{50}q + \tfrac{53}{250}q^2 + \tfrac{3}{50}q^3.$$

A direct-form II realization is

$$w[n] = x[n] - \tfrac{17}{50}w[n-1] - \tfrac{53}{250}w[n-2] - \tfrac{3}{50}w[n-3],$$

$$y[n] = w[n] + \tfrac{4}{5}w[n-1] + \tfrac{17}{100}w[n-2] + \tfrac{1}{100}w[n-3].$$

It uses three shared delays. A first/second-order cascade uses $H_1 = (1 + q/2)/(1 + 3q/10)$ and $H_2 = (1+3q/10+q^2/50)/(1+q/25+q^2/5)$. For any section $(b_0+b_1q+b_2q^2)/(1+a_1q+a_2q^2)$, use $w = x-a_1w_{-1}-a_2w_{-2}$ and $y = b_0w + b_1w_{-1} + b_2w_{-2}$; the first-order case sets $a_2 = b_2 = 0$ and removes its unused delay. Two second-order slots can implement these same sections with a zero second-order coefficient in the first slot. Their unused delay must be omitted to retain canonical three-state storage.

Structural or analytic derivation

To check the degree, factor $1 + 0.3q + 0.02q^2 = (1 + 0.1q)(1 + 0.2q)$, which has no factor in common with the denominator; thus the displayed three-state order is genuine for these factors. The biquad state equations prove each realization by multiplication, and zero padding a coefficient array does not increase the McMillan degree. Reordering sections preserves the exact transfer but changes intermediate signal magnitudes.