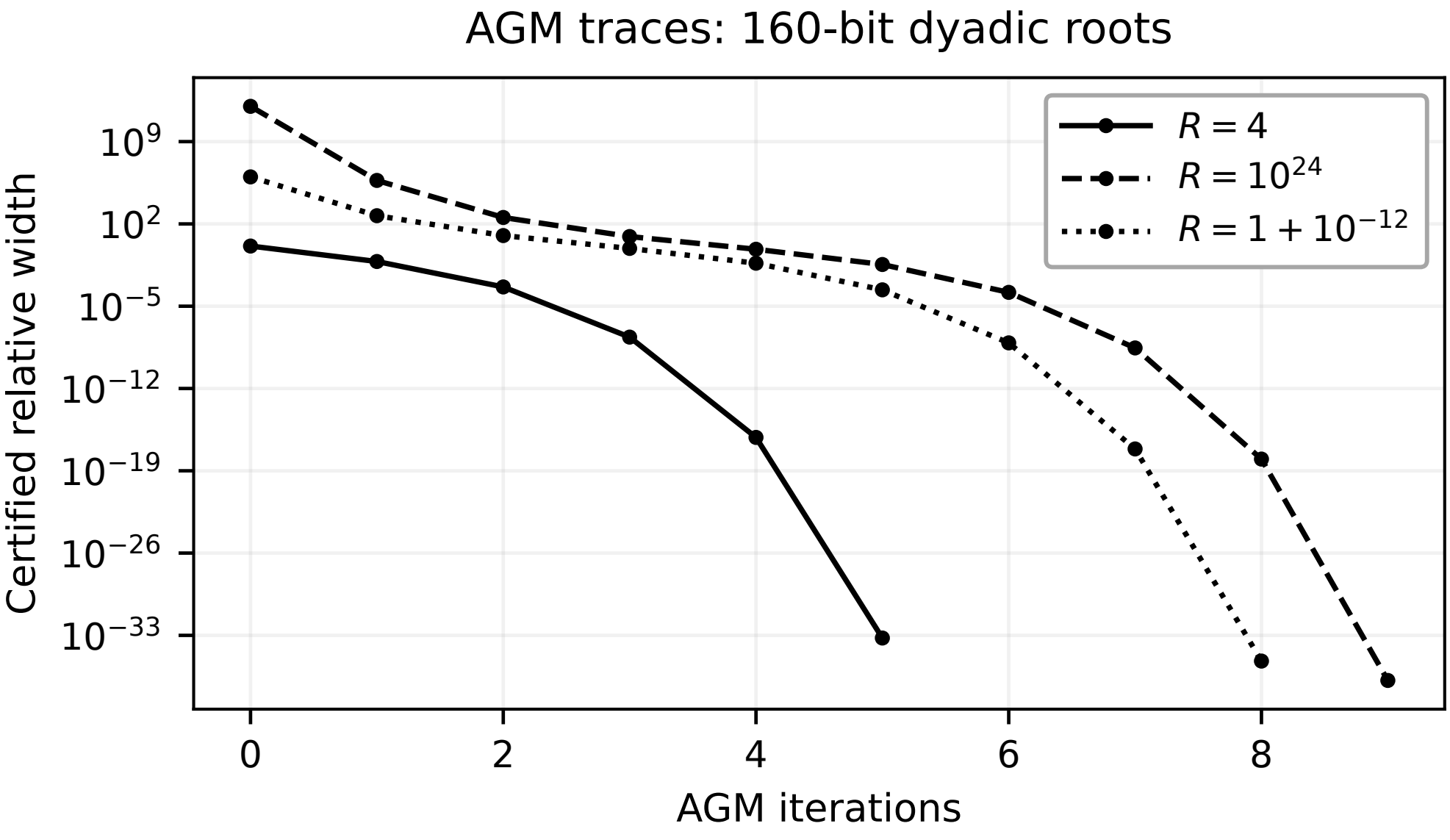
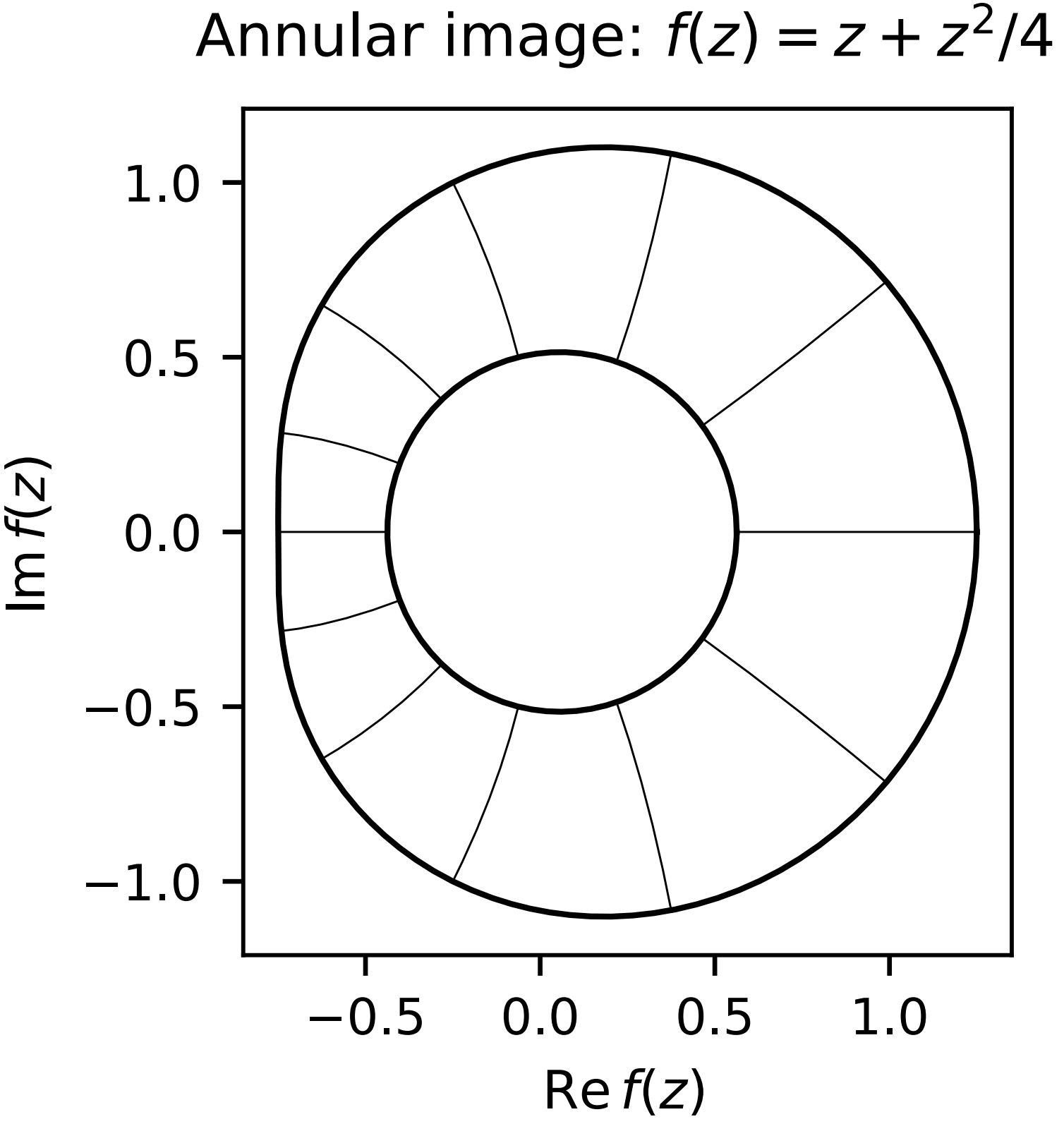
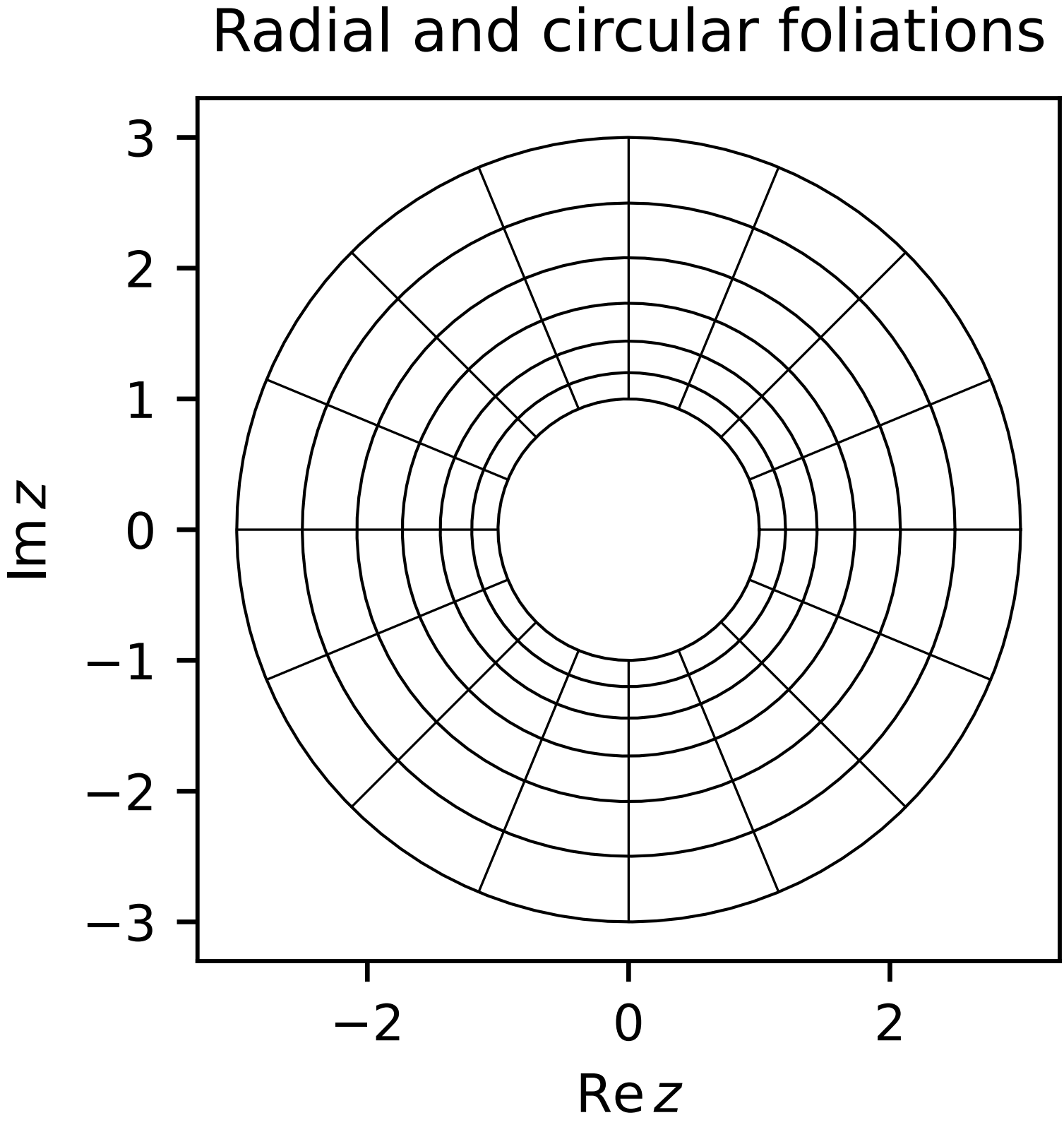
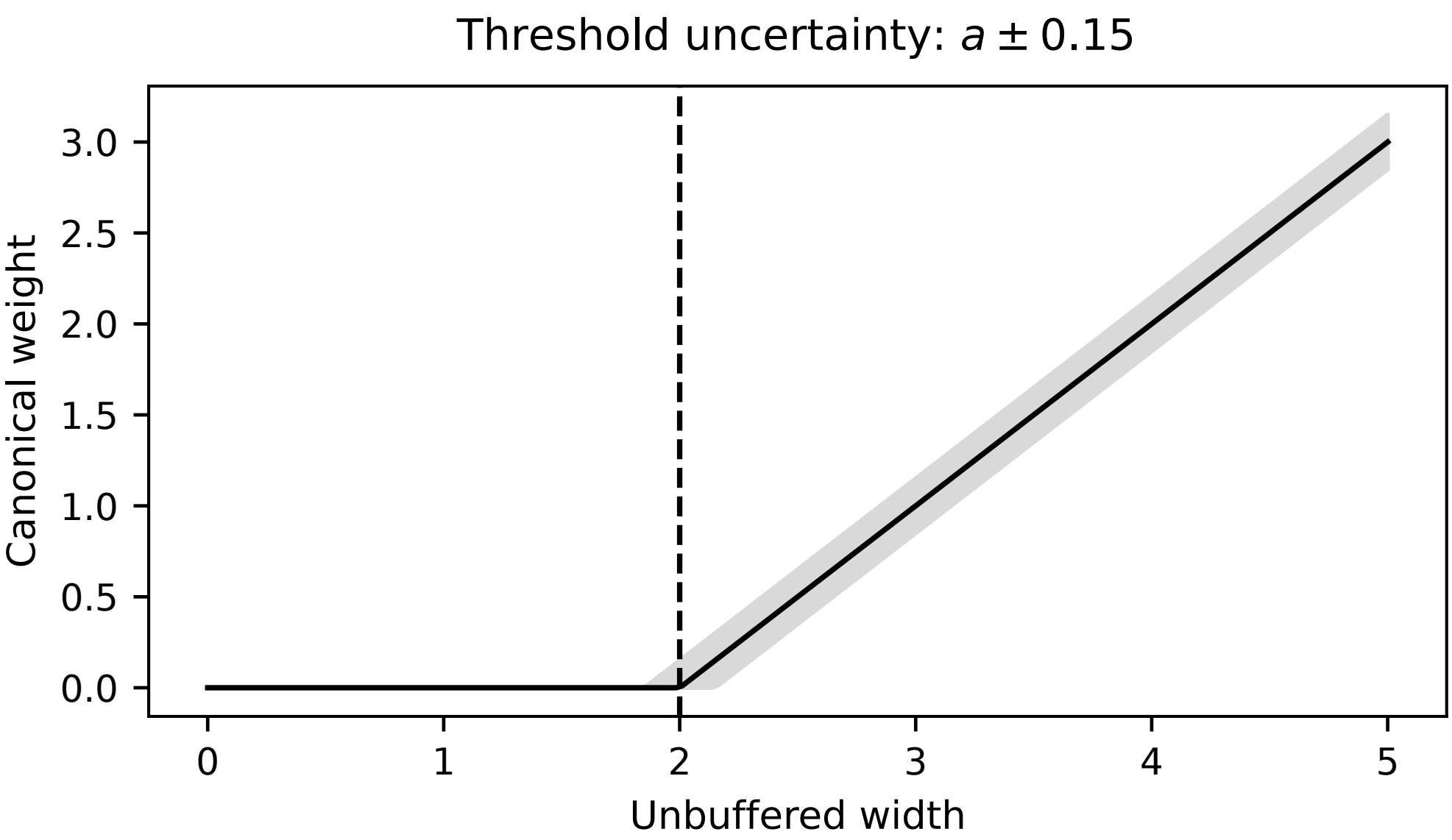
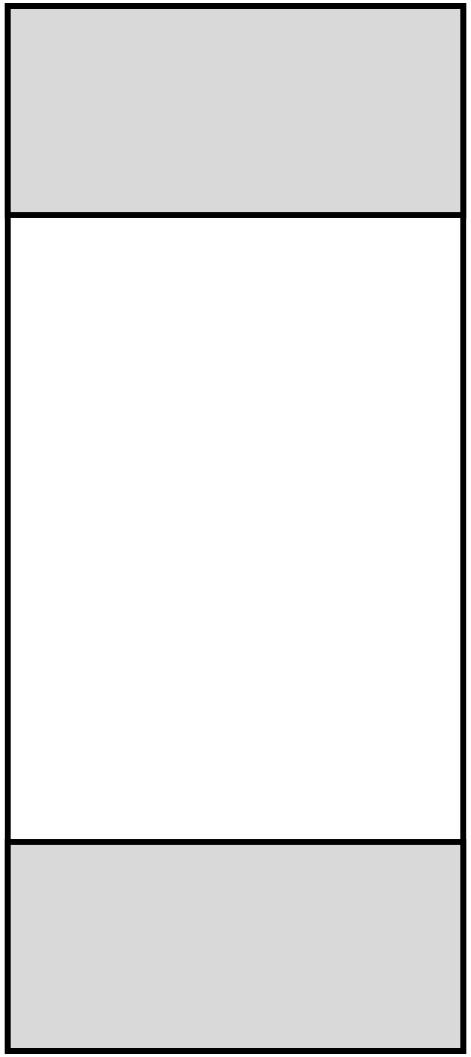
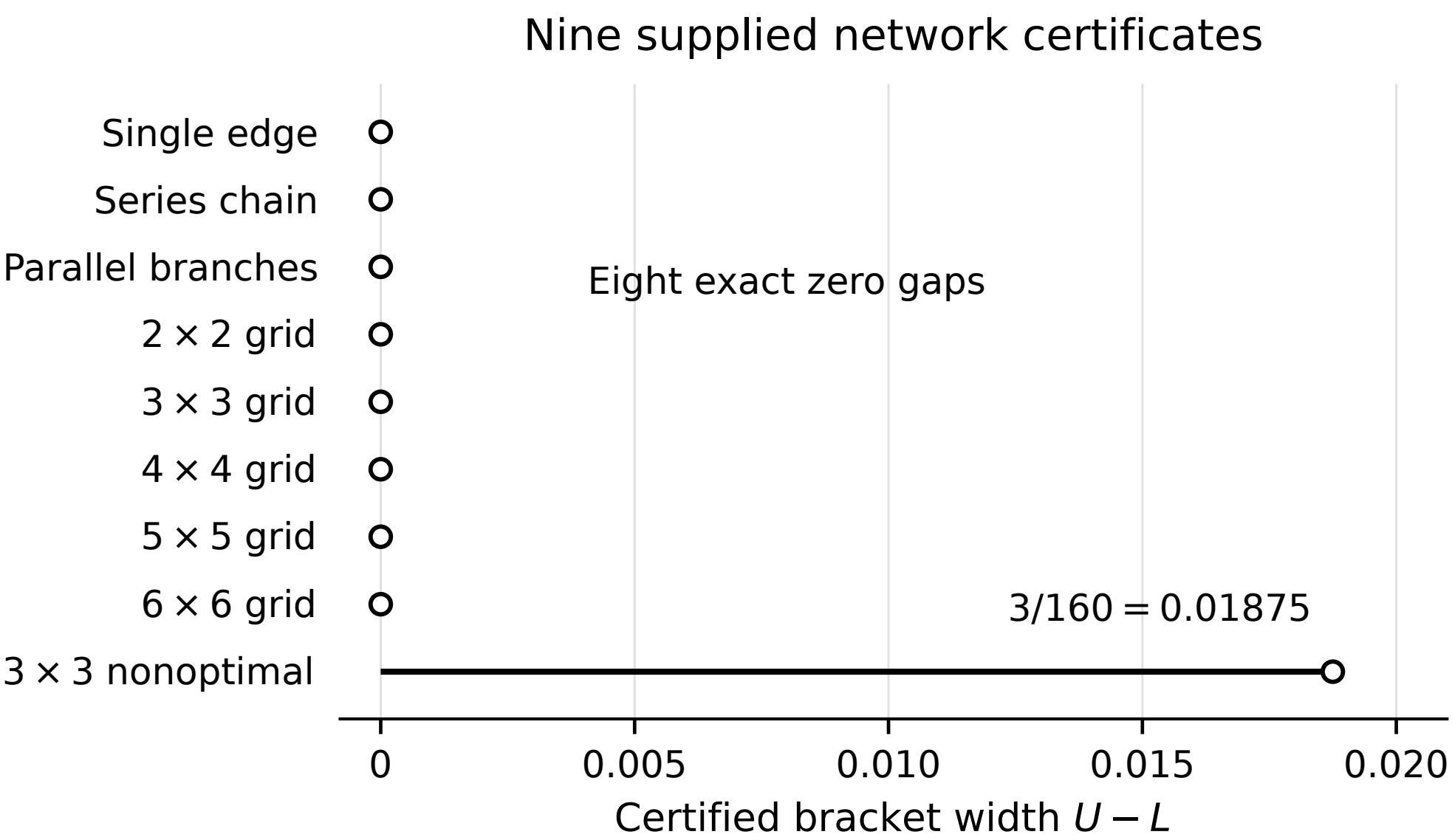

AGM traces: 160-bit dyadic roots
Certified relative width
10^{9}
10^{2}
10^{-5}
10^{-12}
10^{-19}
10^{-26}
10^{-33}
0
2
4
6
8
AGM iterations
$R = 4$
$R = 10^{24}$
$R = 1 + 10^{-12}$

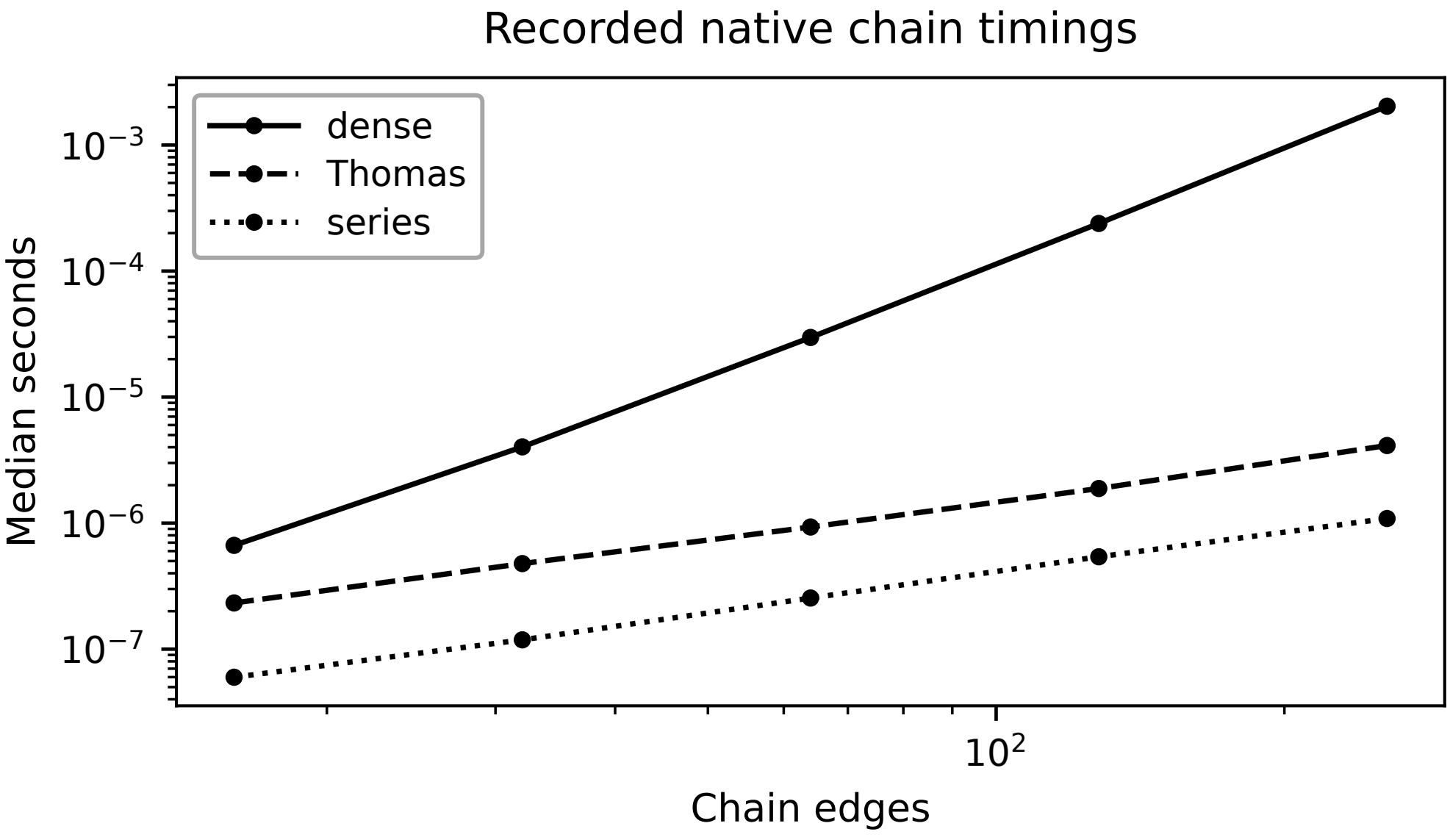

Annular image: $f(z) = z + z^2/4$
1.0
0.5
0.0
−0.5
−1.0
Im $f(z)$
−0.5
0.0
0.5
1.0
Re $f(z)$

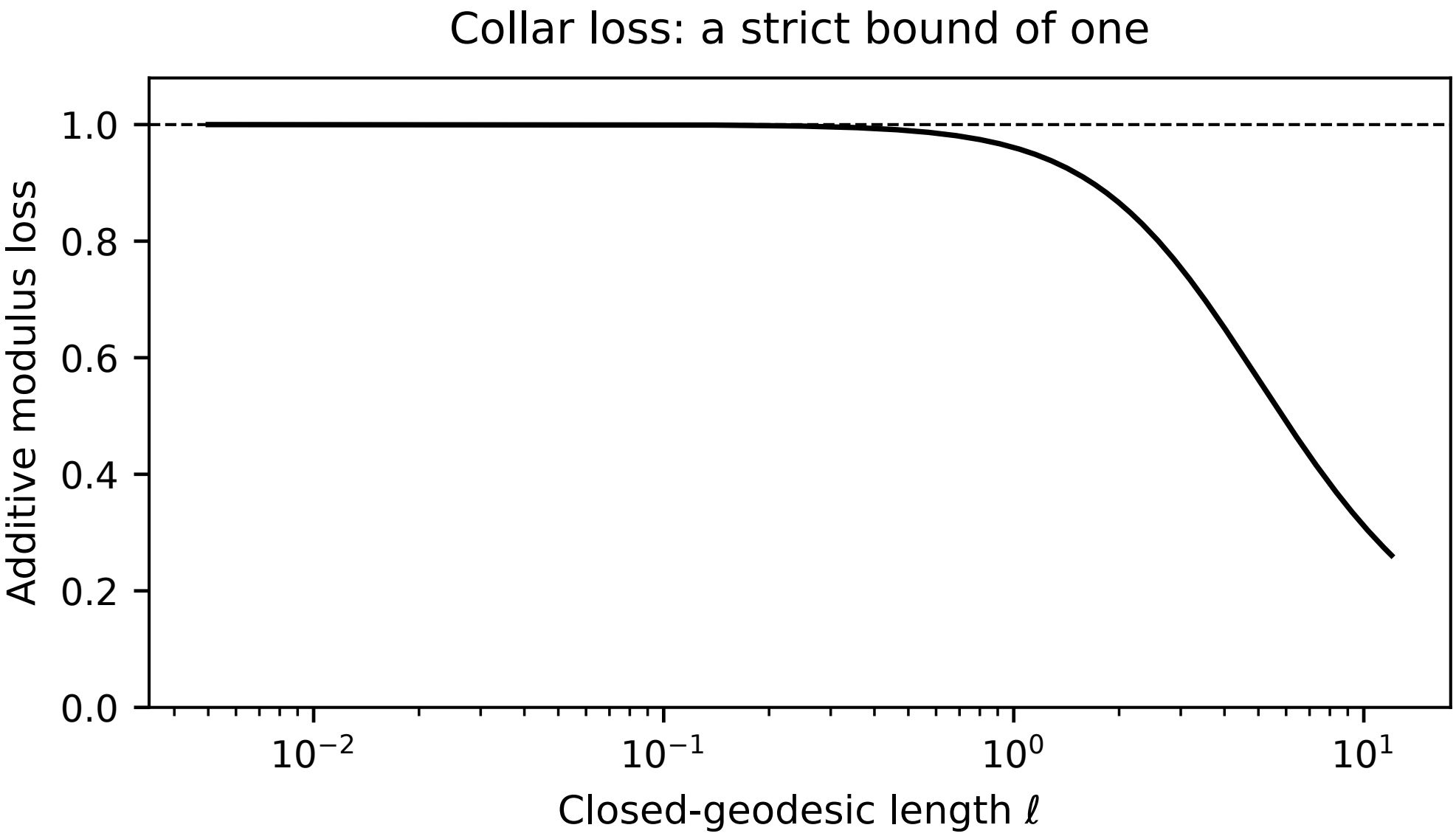

Radial and circular foliations
3
2
1
0
−1
−2
−3
Im z
−2
0
2
Re z

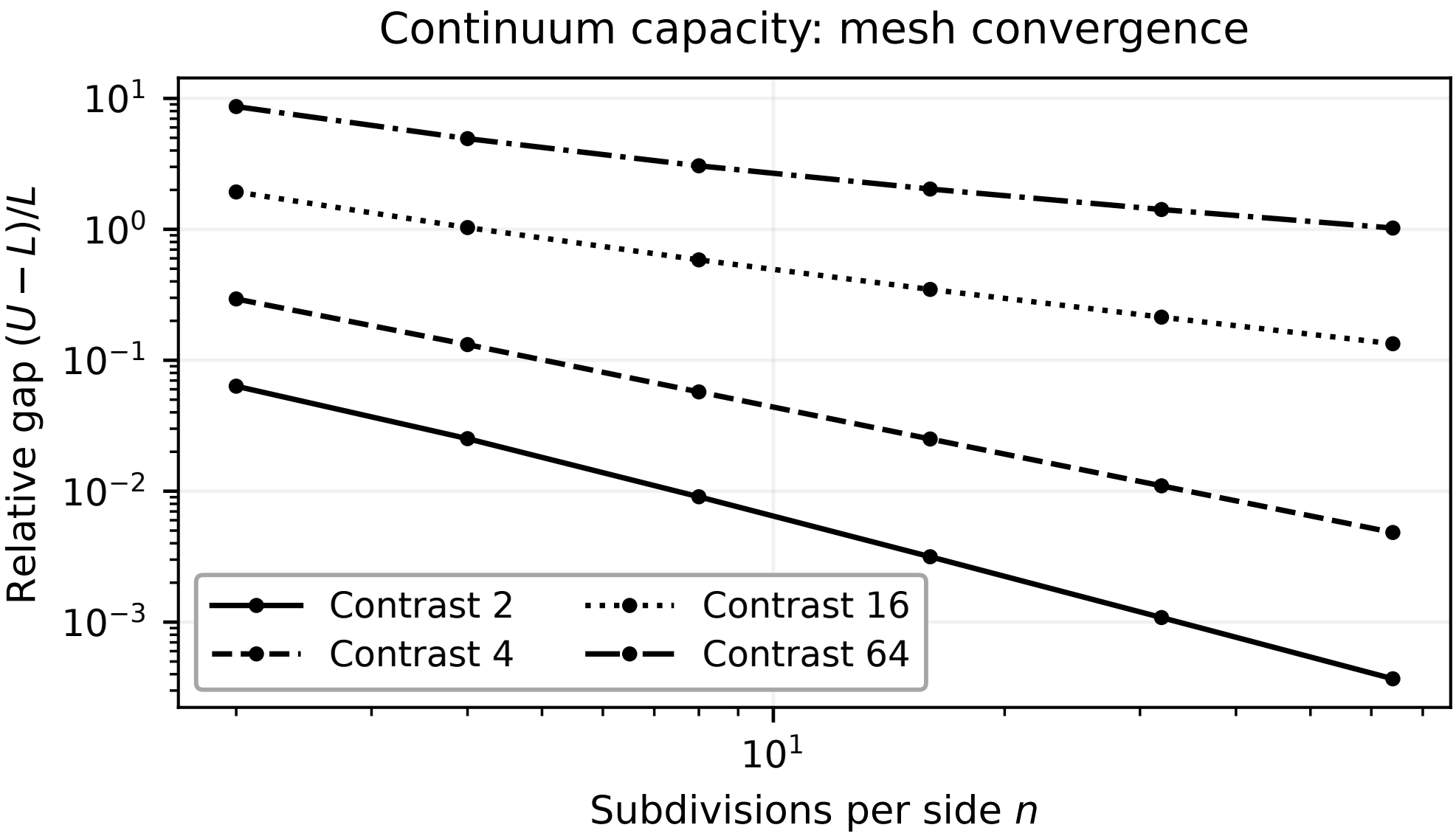

Threshold uncertainty: $a \pm 0.15$
Canonical weight
3.0
2.5
2.0
1.5
1.0
0.5
0.0
0
1
2
3
4
5
Unbuffered width

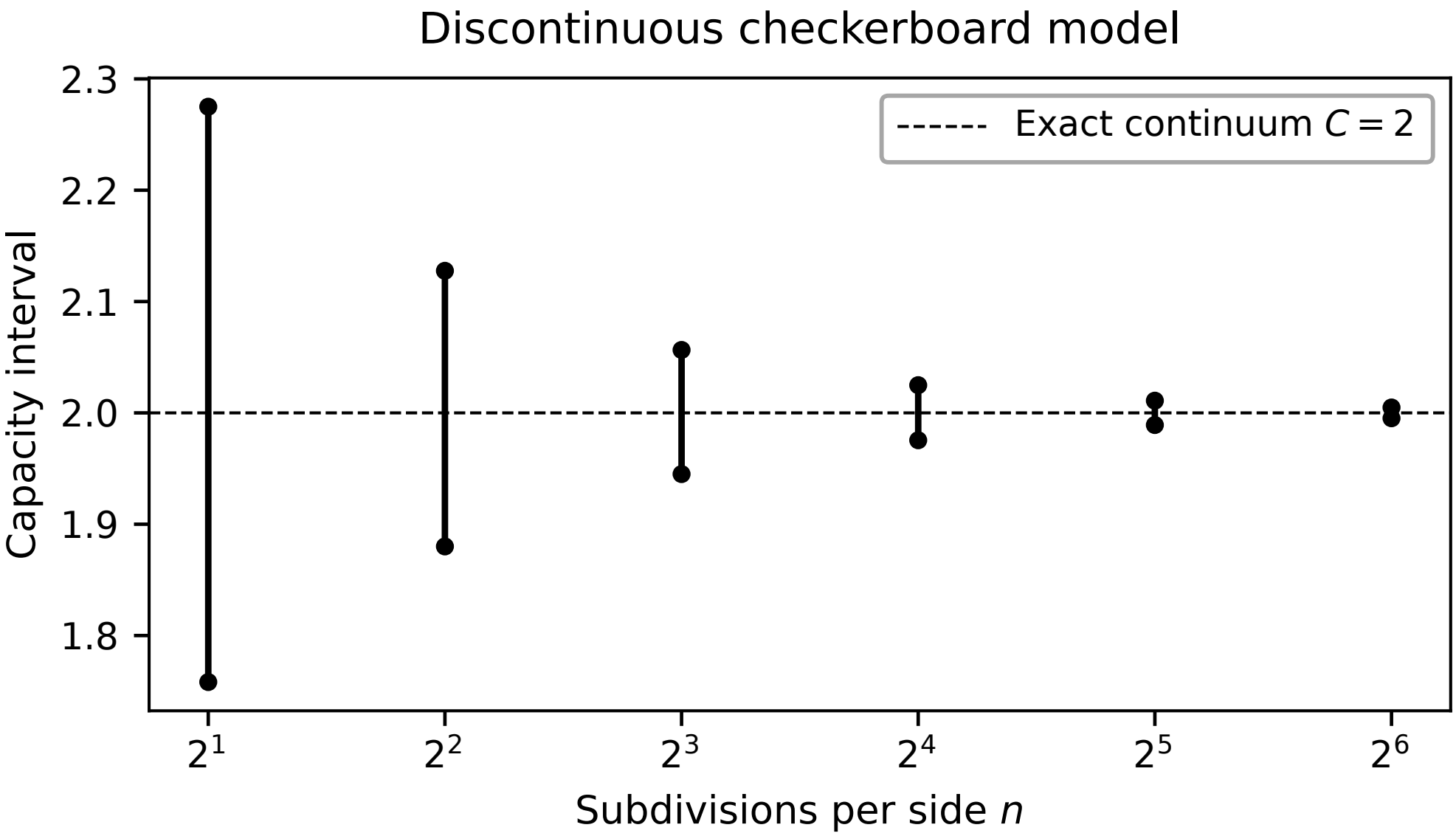

Upper buffer M
Remaining height $h - 2M$
Lower buffer M

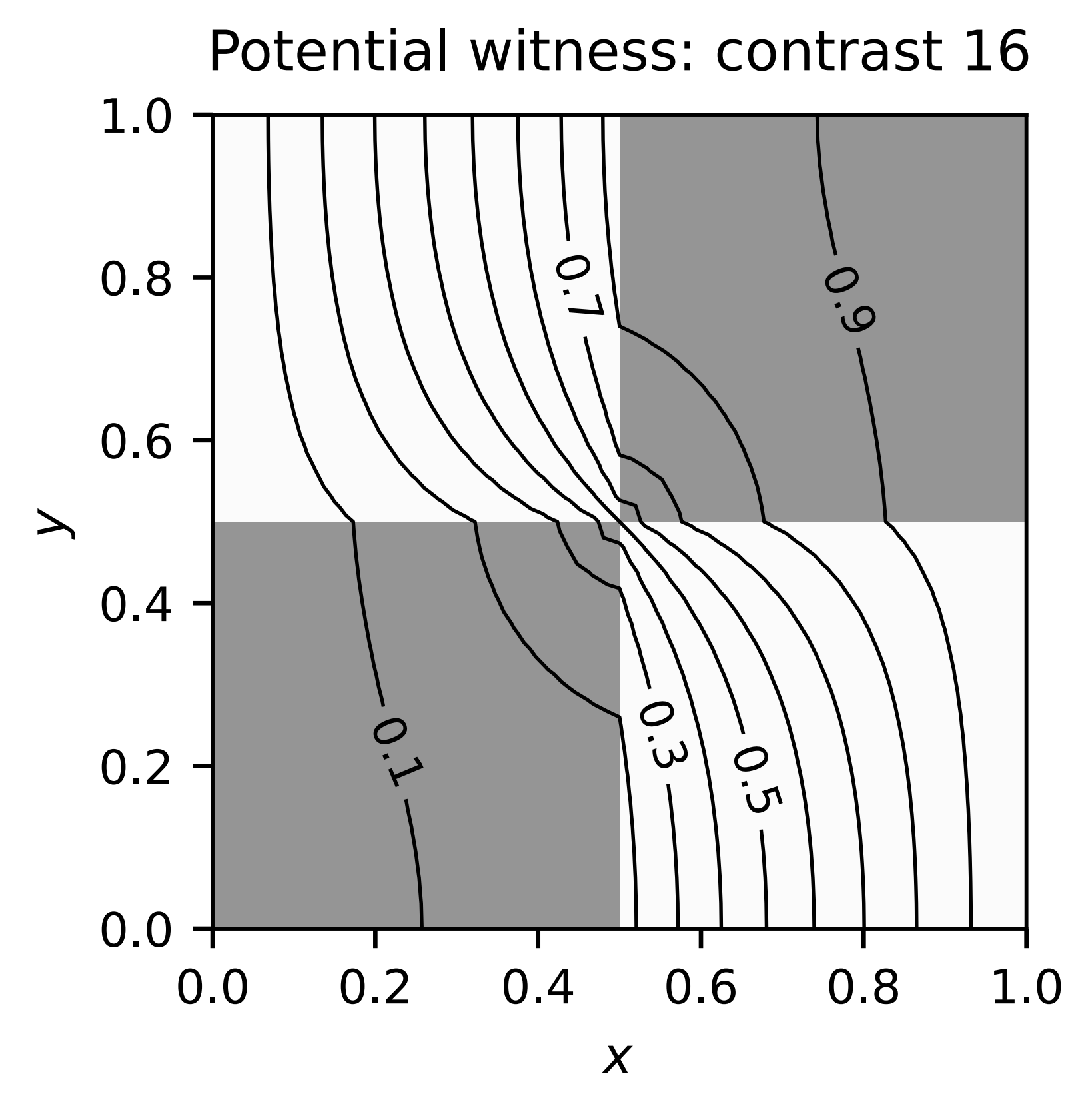

Nine supplied network certificates
Single edge
Series chain
Parallel branches
2 × 2 grid
3 × 3 grid
4 × 4 grid
5 × 5 grid
6 × 6 grid
3 × 3 nonoptimal
Eight exact zero gaps
3/160 = 0.01875
0
0.005
0.010
0.015
0.020
Certified bracket width $U - L$

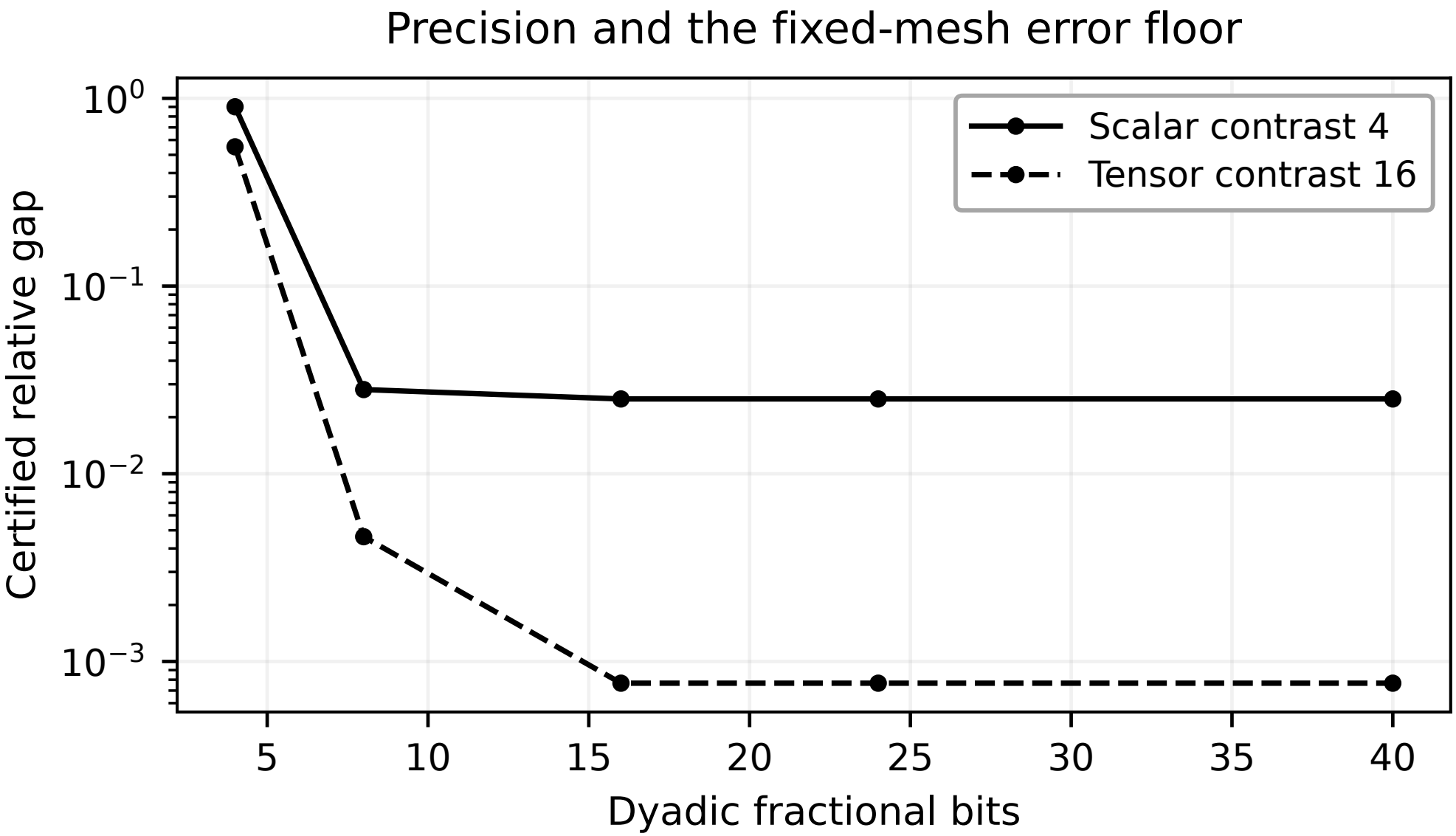

Recorded native chain timings
dense
Thomas
series
Median seconds
10^{-3}
10^{-4}
10^{-5}
10^{-6}
10^{-7}
10^{2}
Chain edges

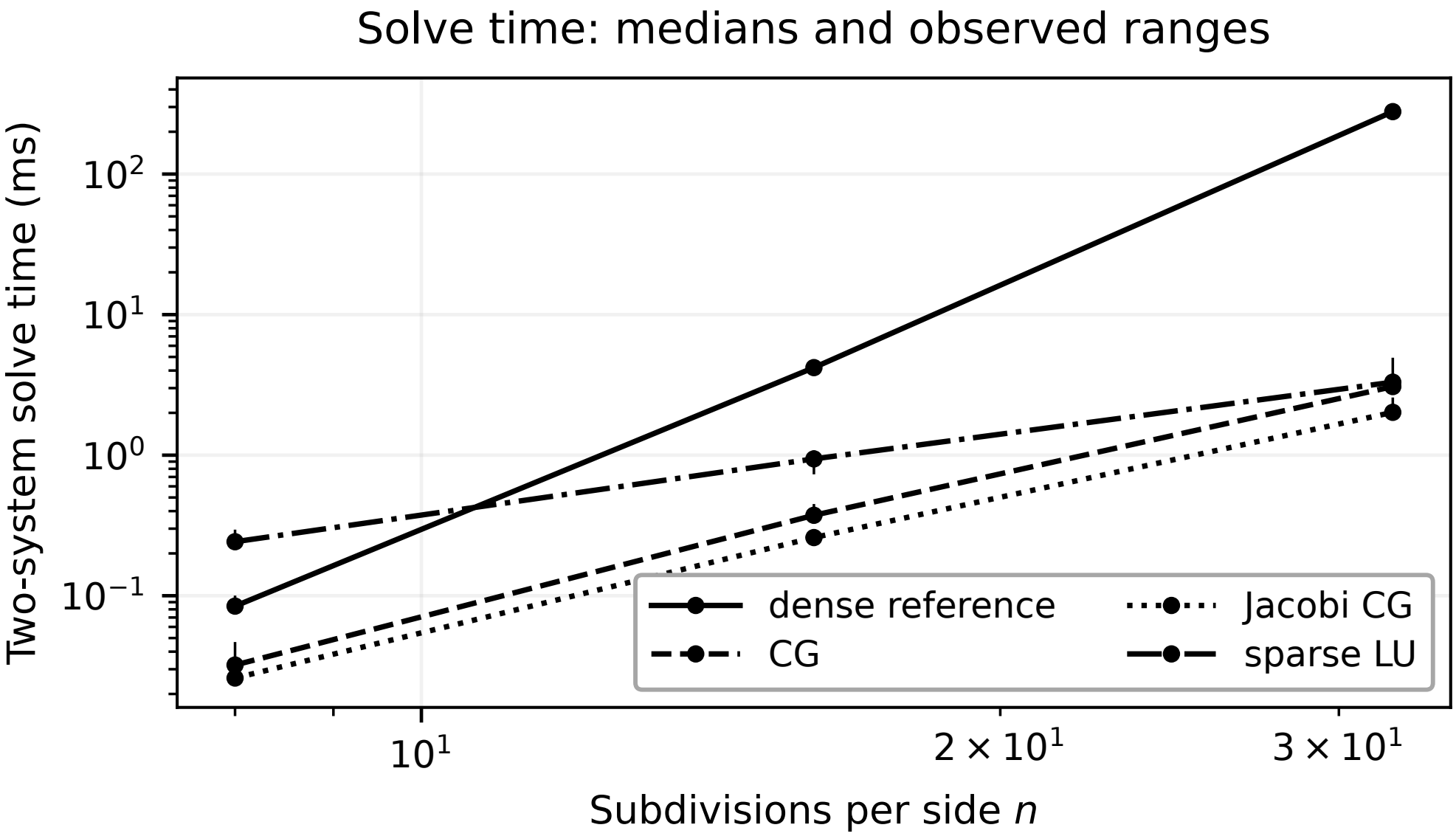

Collar loss: a strict bound of one
Additive modulus loss
1.0
0.8
0.6
0.4
0.2
0.0
10^{-2}
10^{-1}
10^{0}
10^{1}
Closed-geodesic length ℓ

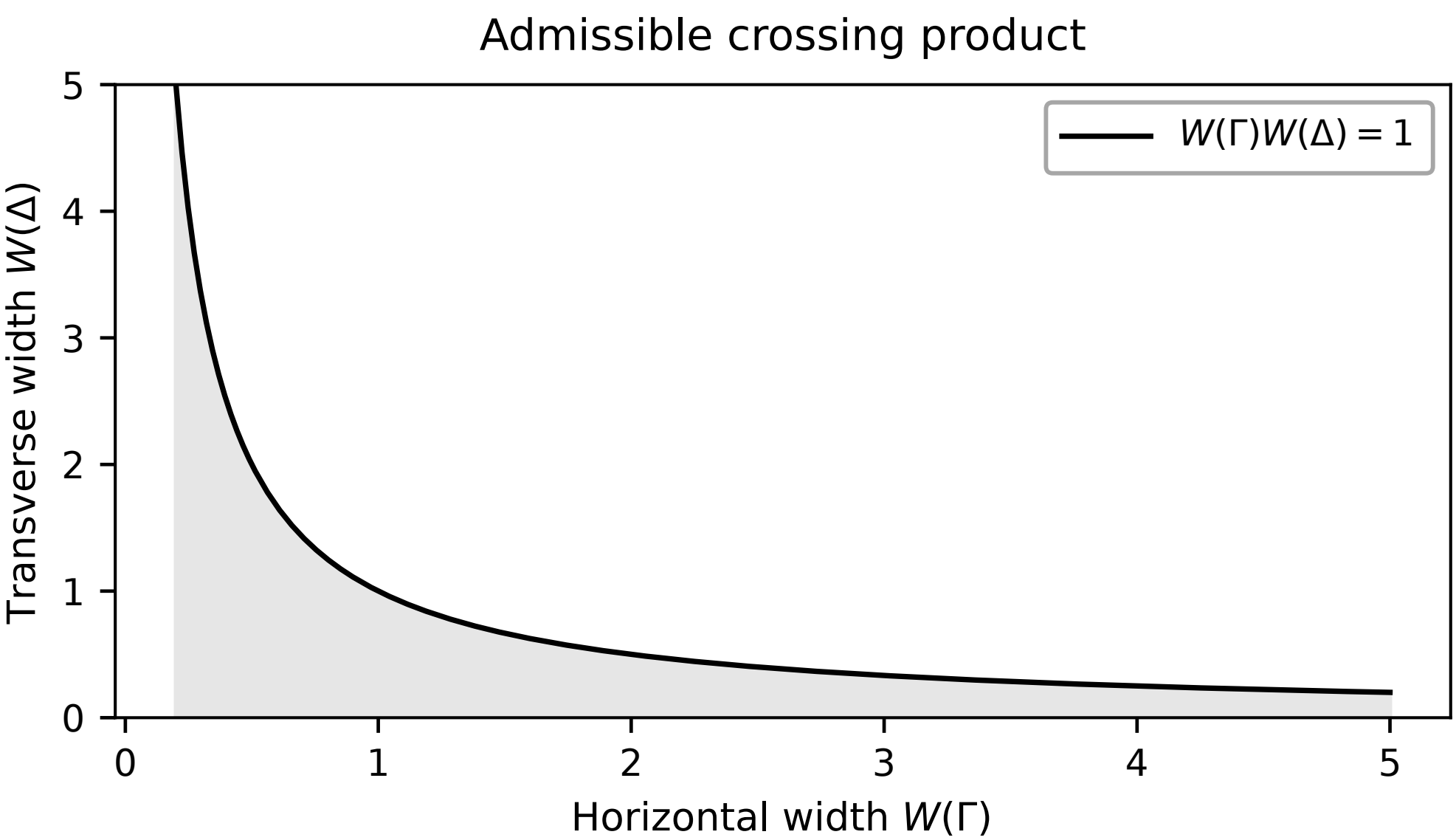

Continuum capacity: mesh convergence
Relative gap $(U-L)/L$
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{1}
Subdivisions per side n
Contrast 2
Contrast 4
Contrast 16
Contrast 64

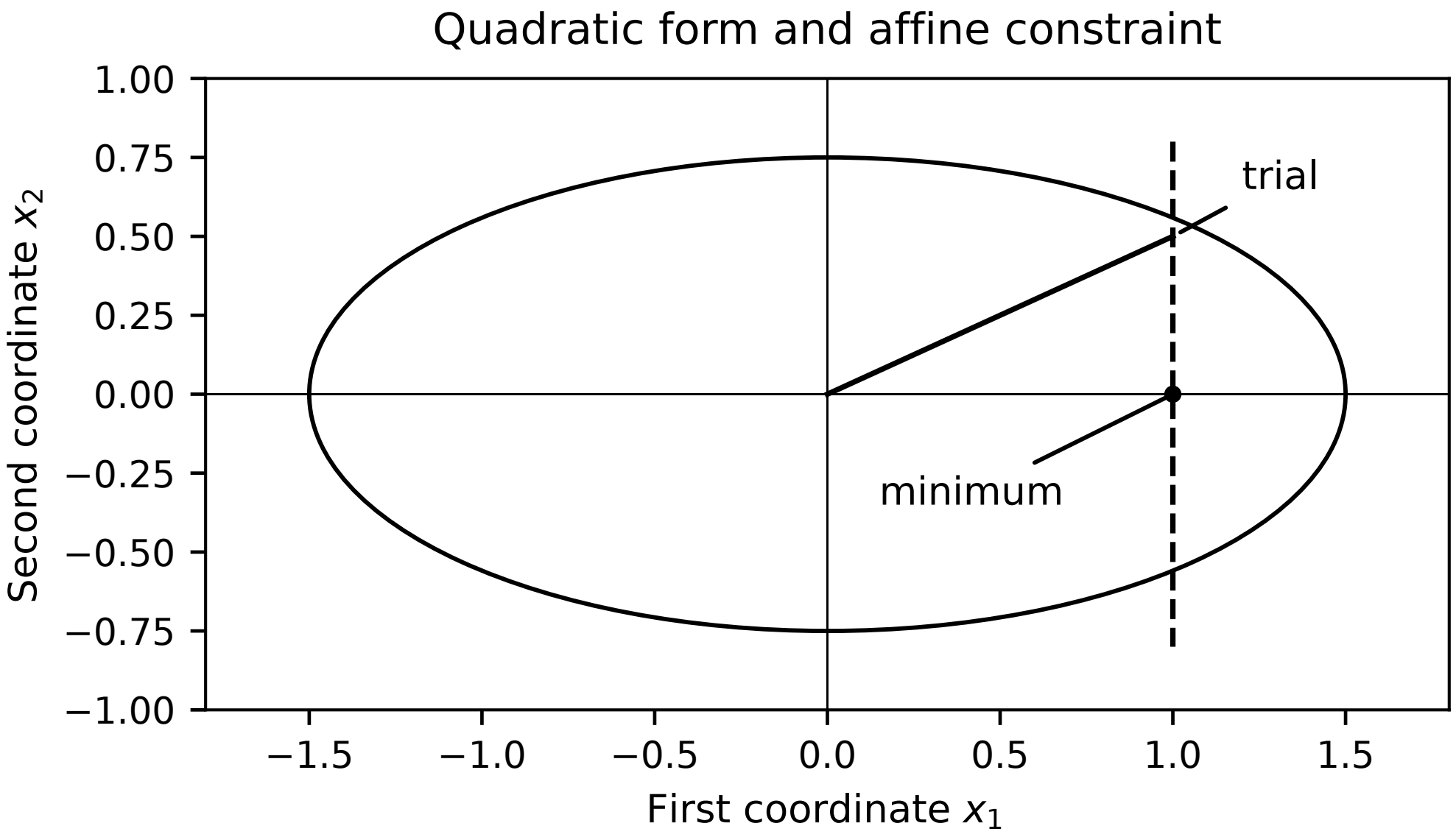

Discontinuous checkerboard model
Exact continuum $C = 2$
Capacity interval
2.3
2.2
2.1
2.0
1.9
1.8
2^1
2^2
2^3
2^4
2^5
2^6
Subdivisions per side n

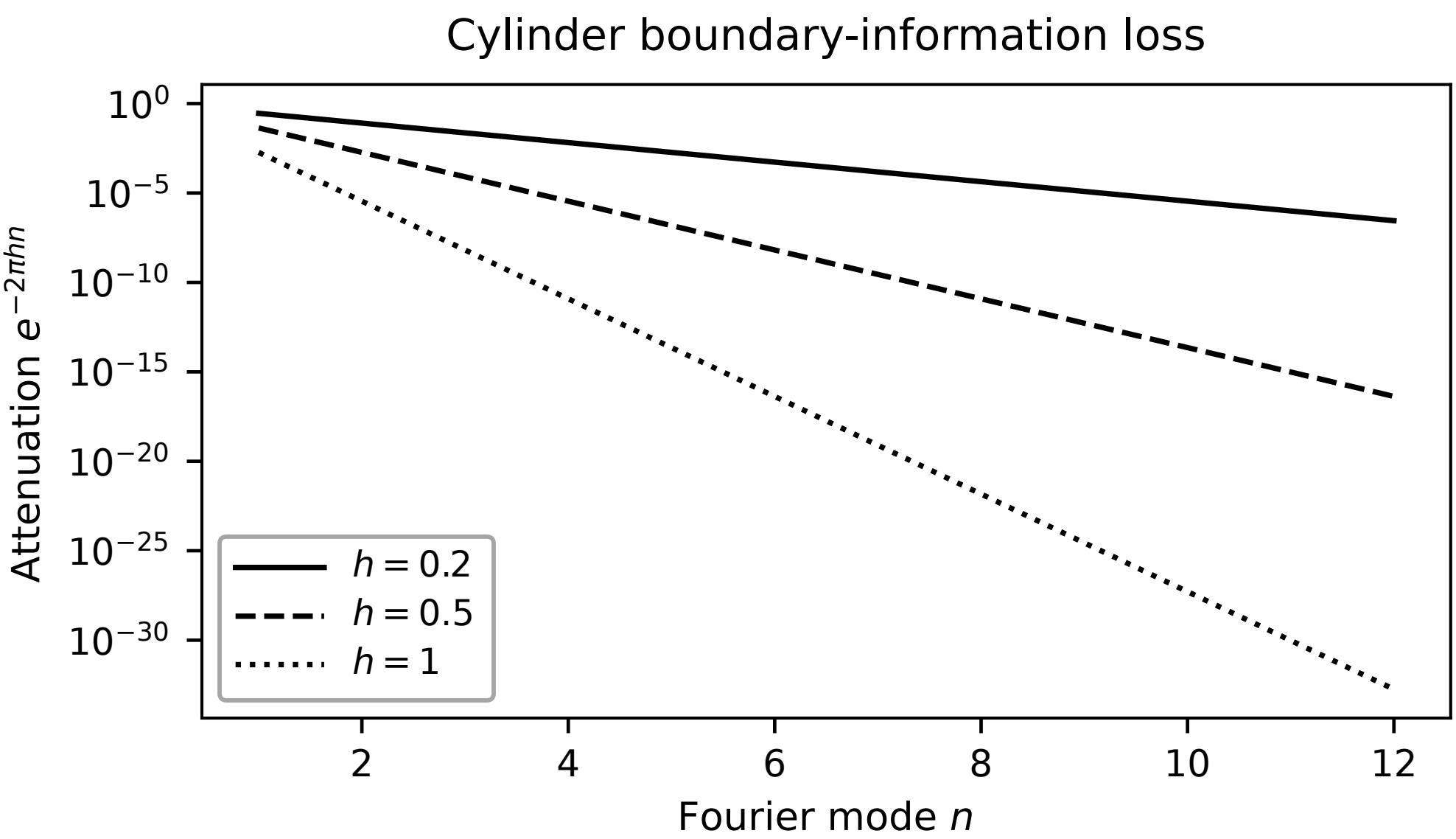

Potential witness: contrast 16
1.0
0.8
0.6
0.4
0.2
0.0
y
0.0
0.2
0.4
0.6
0.8
1.0
x
0.1
0.3
0.5
0.7
0.9

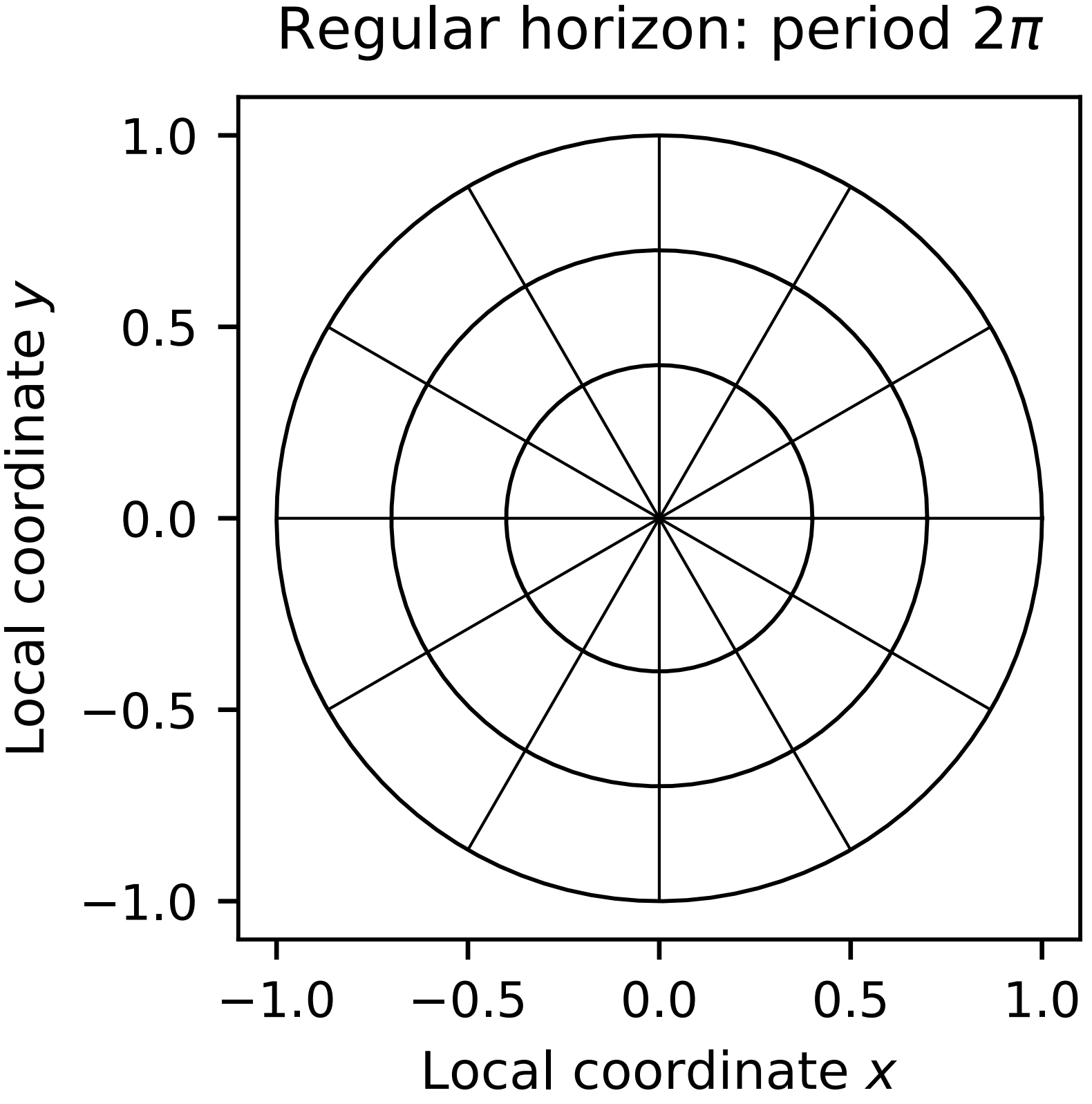

Precision and the fixed-mesh error floor
Certified relative gap
10^{0}
10^{-1}
10^{-2}
10^{-3}
5
10
15
20
25
30
35
40
Dyadic fractional bits
Scalar contrast 4
Tensor contrast 16


Solve time: medians and observed ranges
Two-system solve time (ms)
10^2
10^1
10^0
10^{-1}
10^1
2×10^1
3×10^1
Subdivisions per side n
dense reference
CG
Jacobi CG
sparse LU

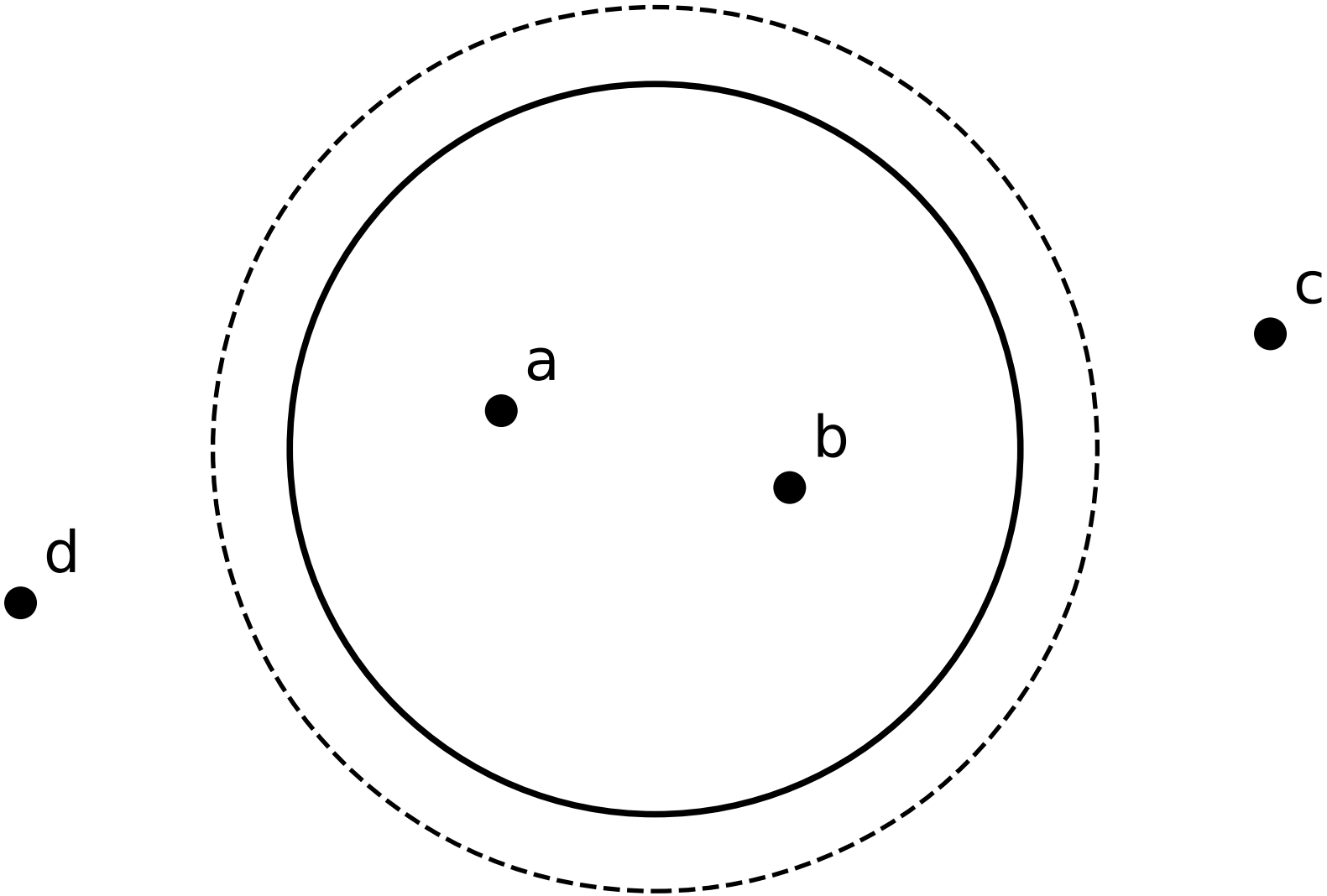

Admissible crossing product
$W(\Gamma)W(\Delta) = 1$
Transverse width $W(\Delta)$
Horizontal width $W(\Gamma)$
0
1
2
3
4
5

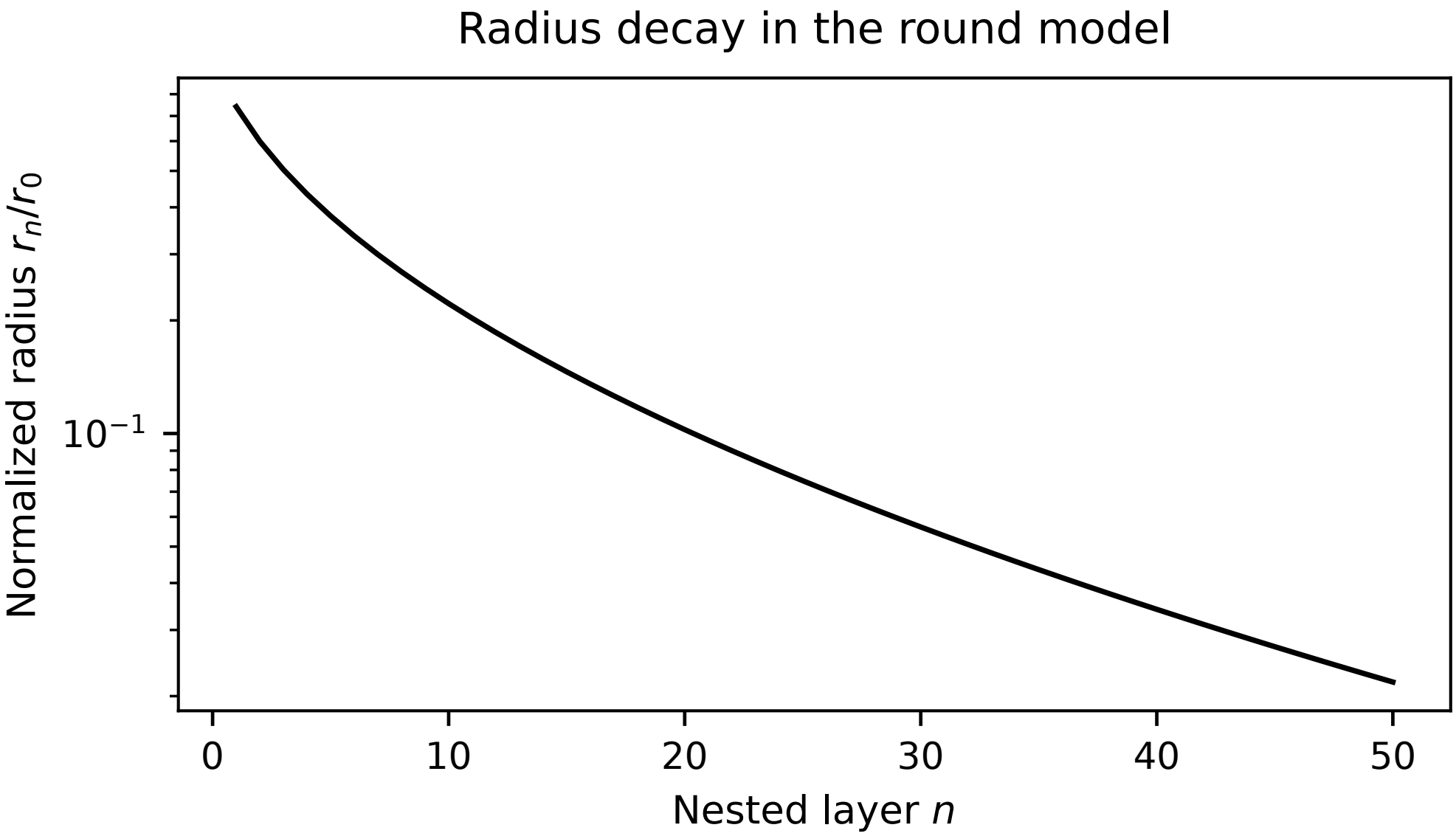

Quadratic form and affine constraint
1.00
0.75
0.50
0.25
0.00
−0.25
−0.50
−0.75
−1.00
Second coordinate x_2
−1.5
−1.0
−0.5
0.0
0.5
1.0
1.5
First coordinate x_1
trial
minimum

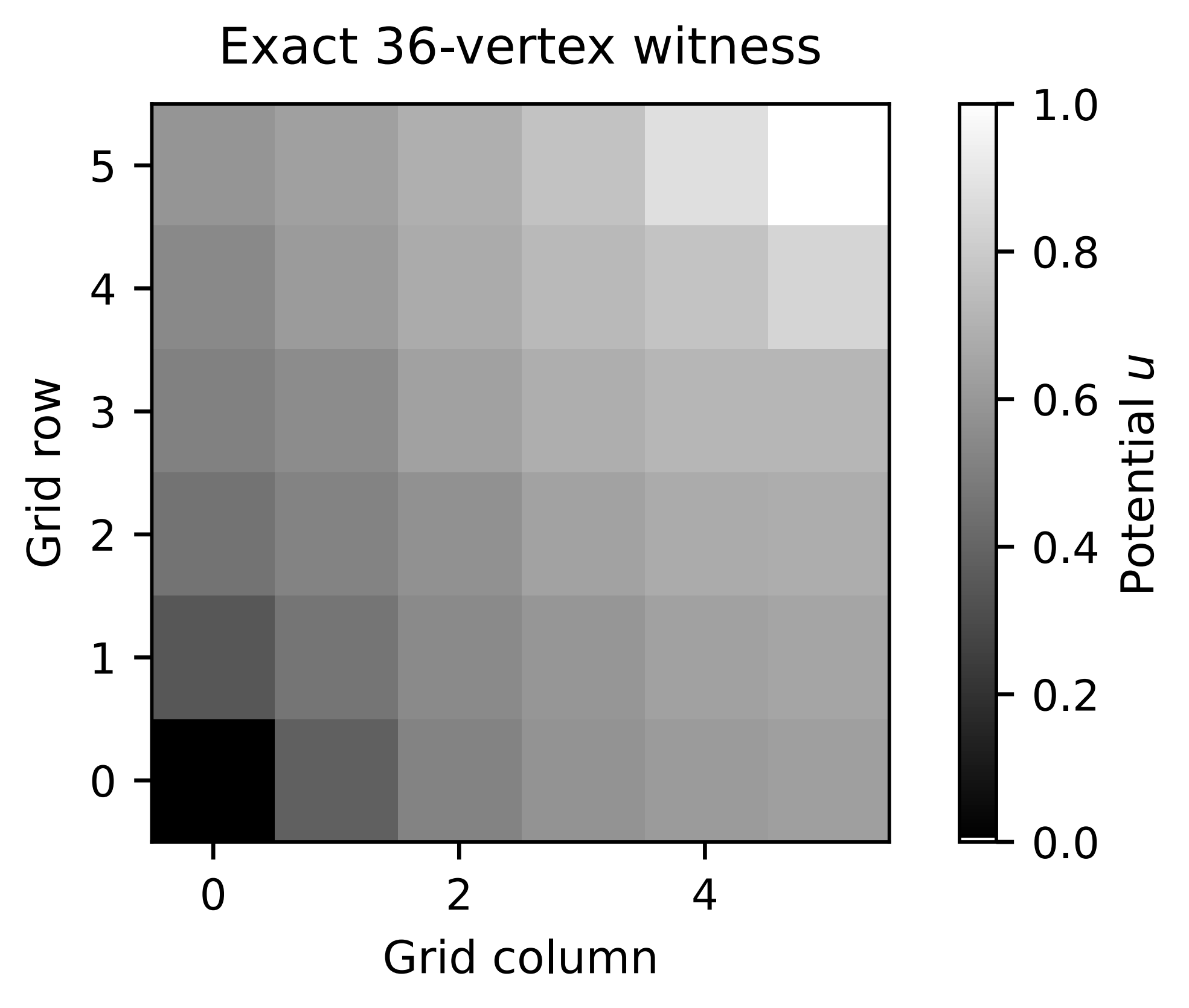

Cylinder boundary-information loss
Attenuation $e^{-2\pi hn}$
10^{0}
10^{-5}
10^{-10}
10^{-15}
10^{-20}
10^{-25}
10^{-30}
$h = 0.2$
$h = 0.5$
$h = 1$
2
4
6
8
10
12
Fourier mode n

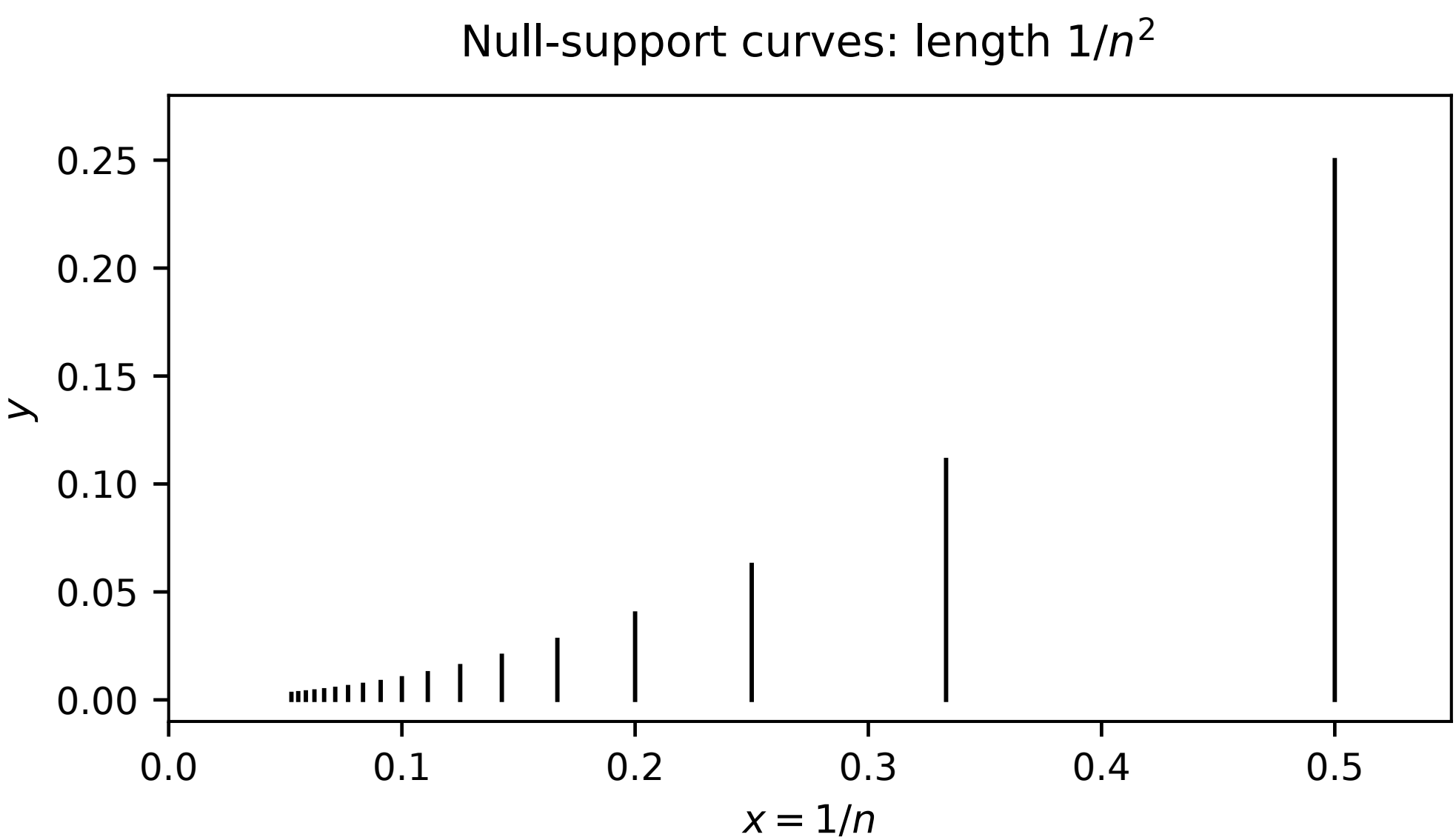

Regular horizon: period 2π
Local coordinate y
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
Local coordinate x

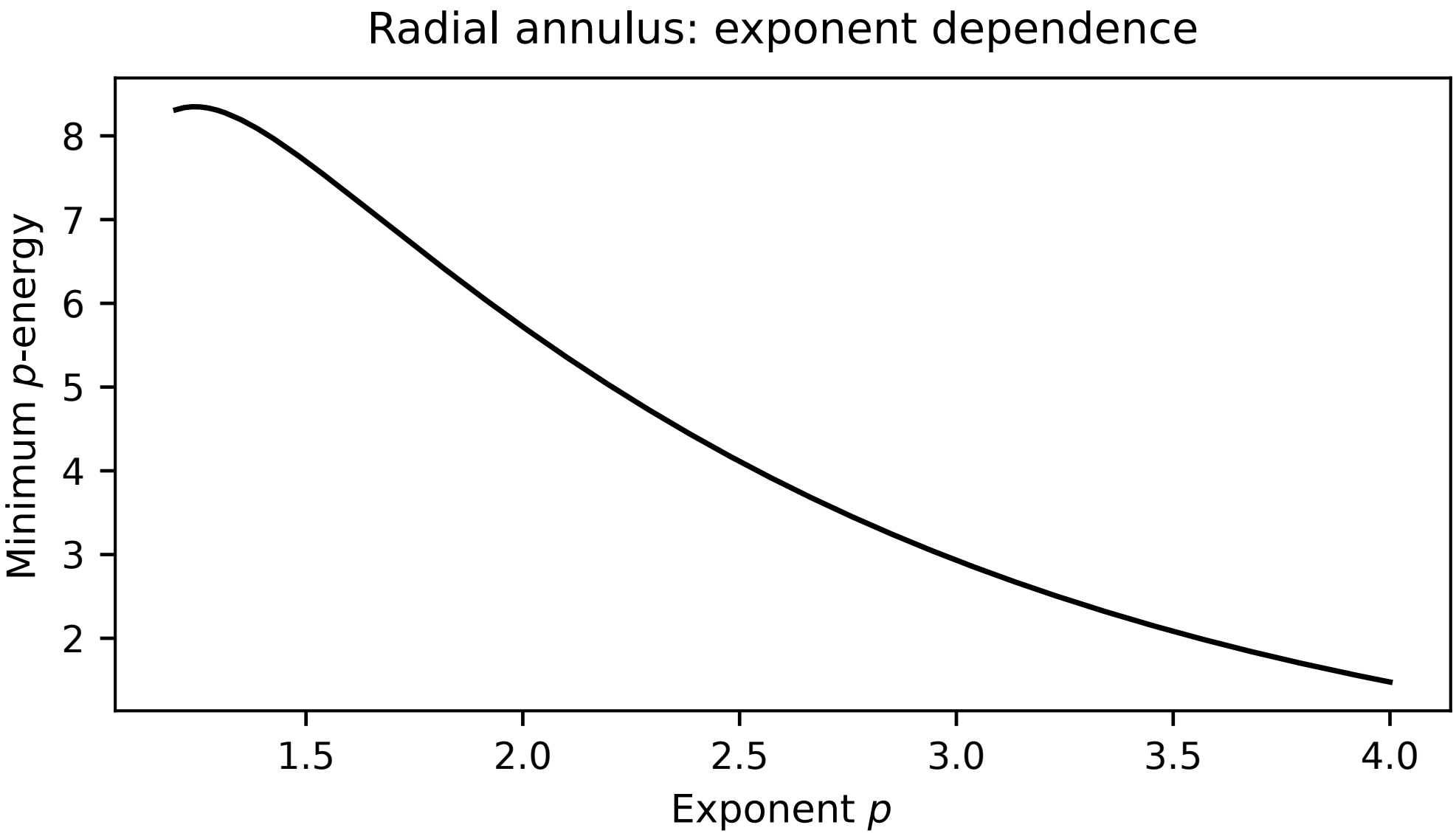
A separating loop cannot collapse

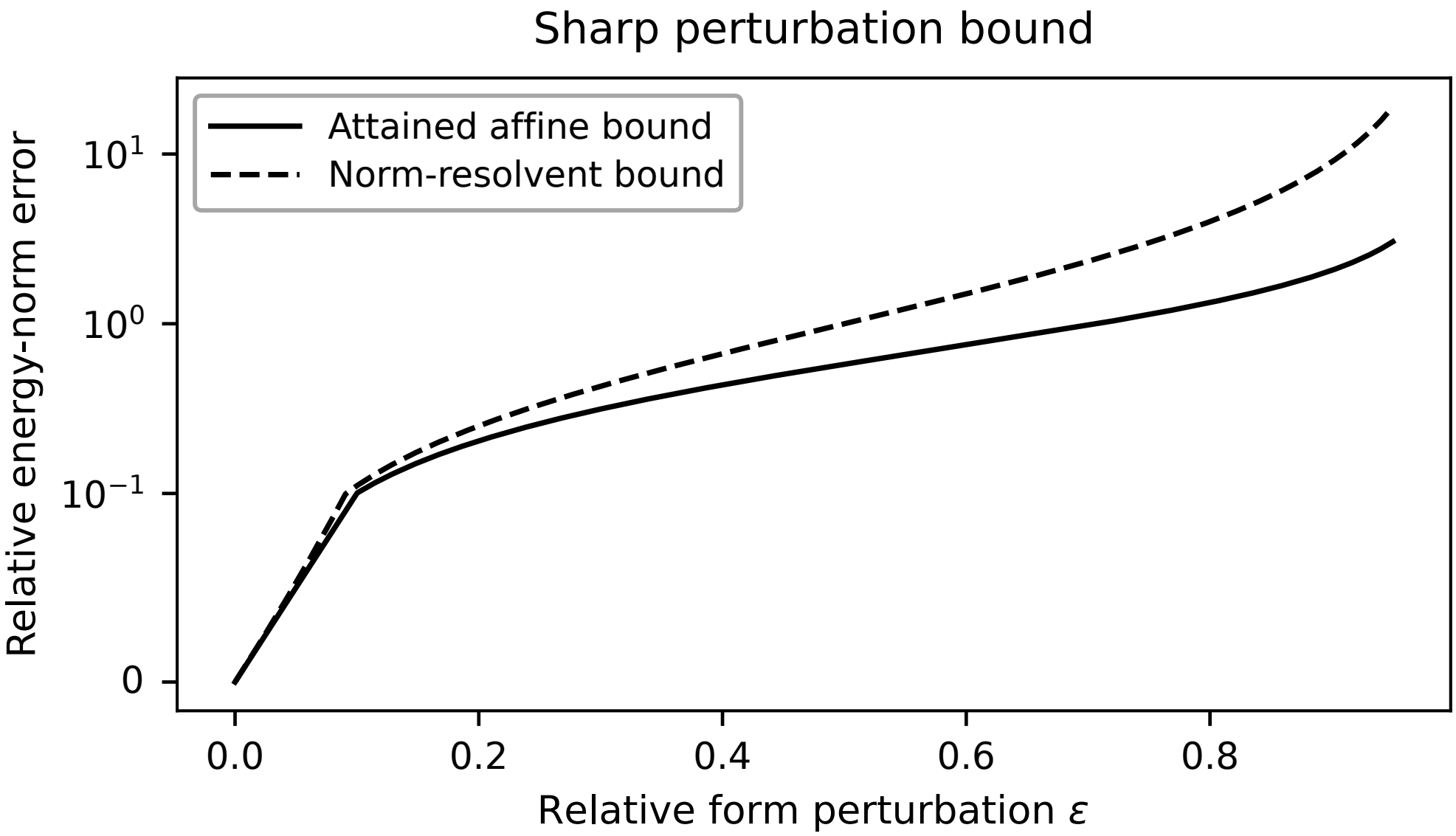


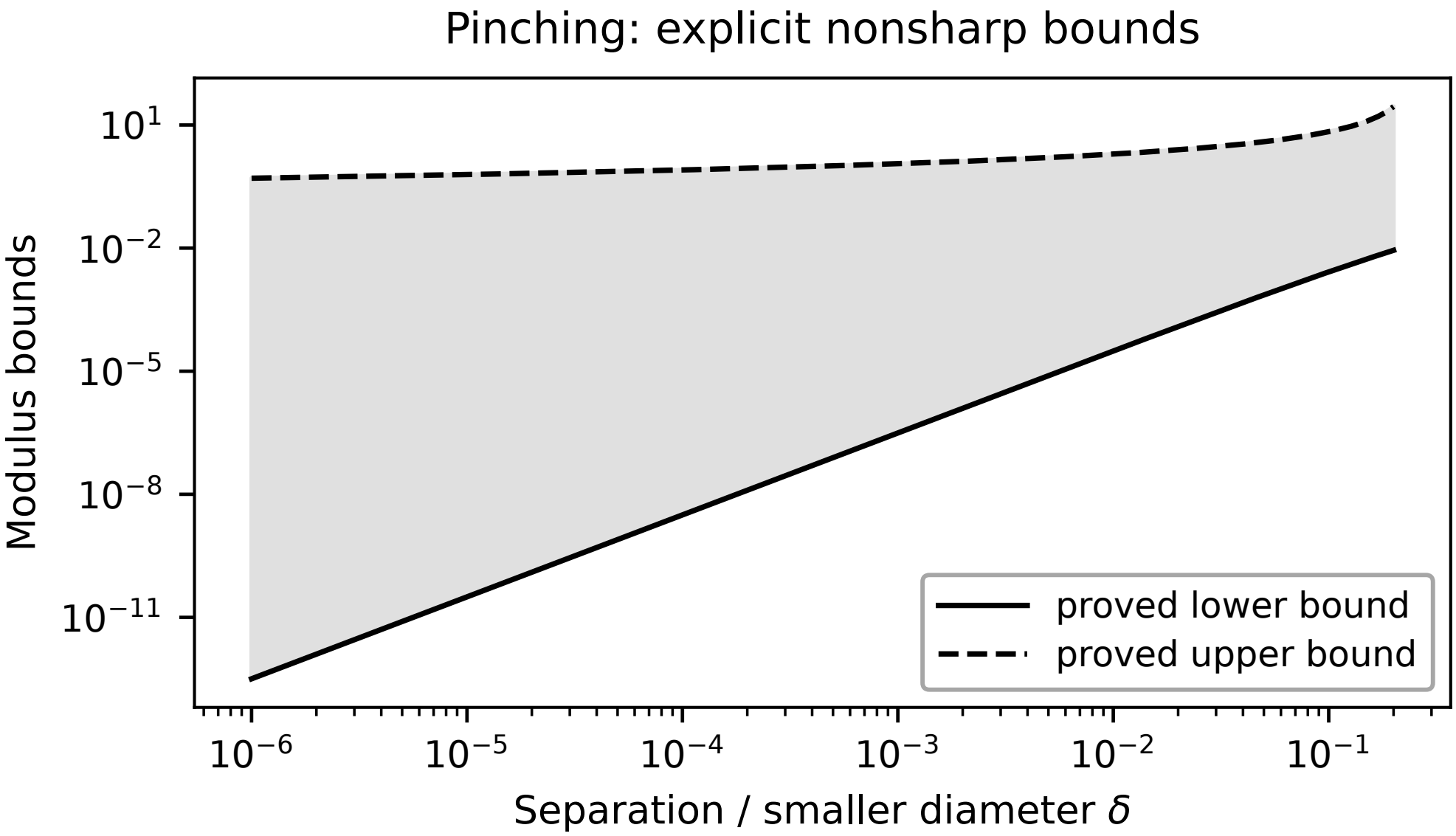

Radius decay in the round model
Normalized radius r_n/r_0
10^{-1}
0
10
20
30
40
50
Nested layer n

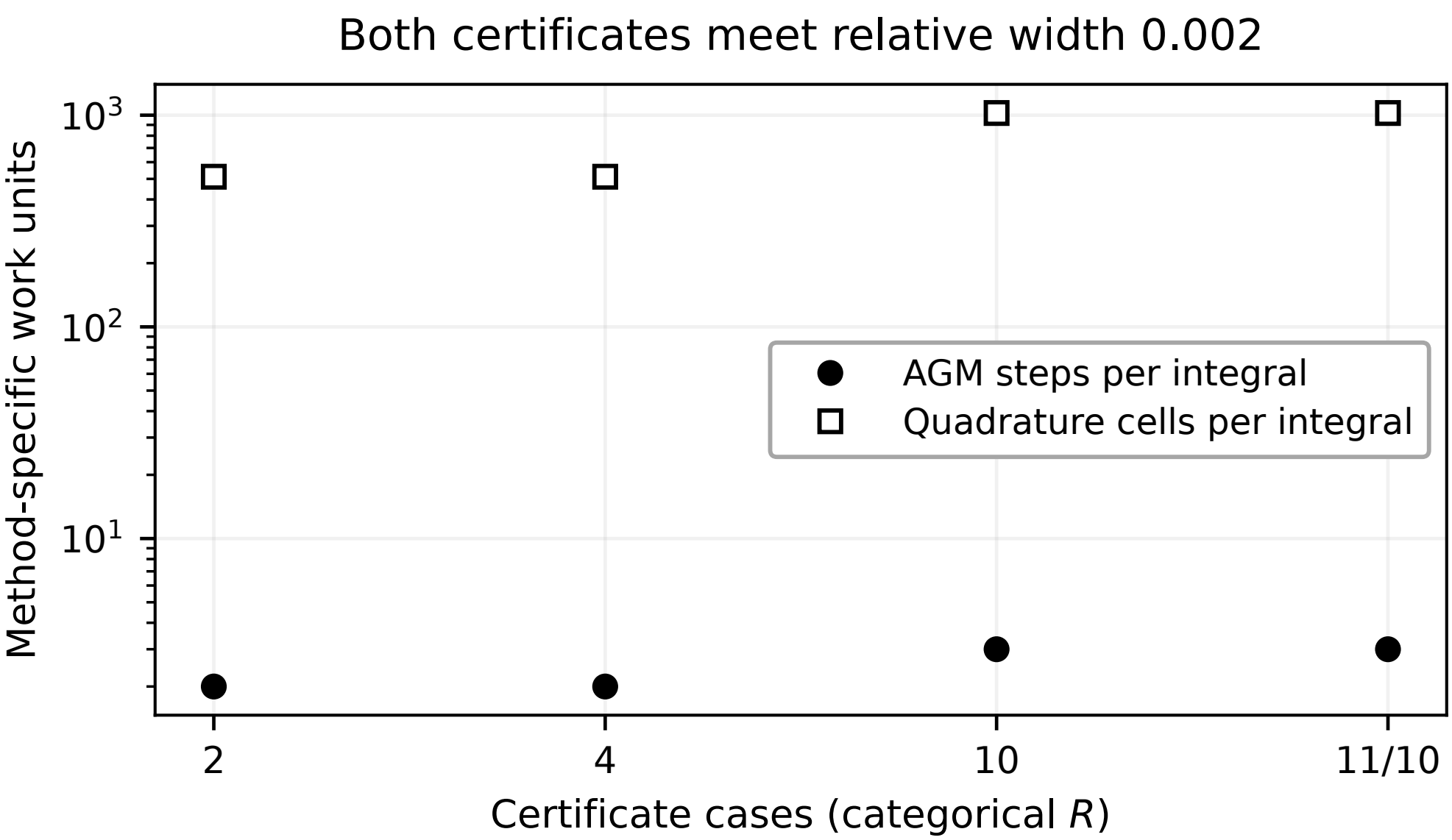

Exact 36-vertex witness
Grid row
Grid column
Potential u
0
1
2
3
4
5
0
2
4
0.0
0.2
0.4
0.6
0.8
1.0

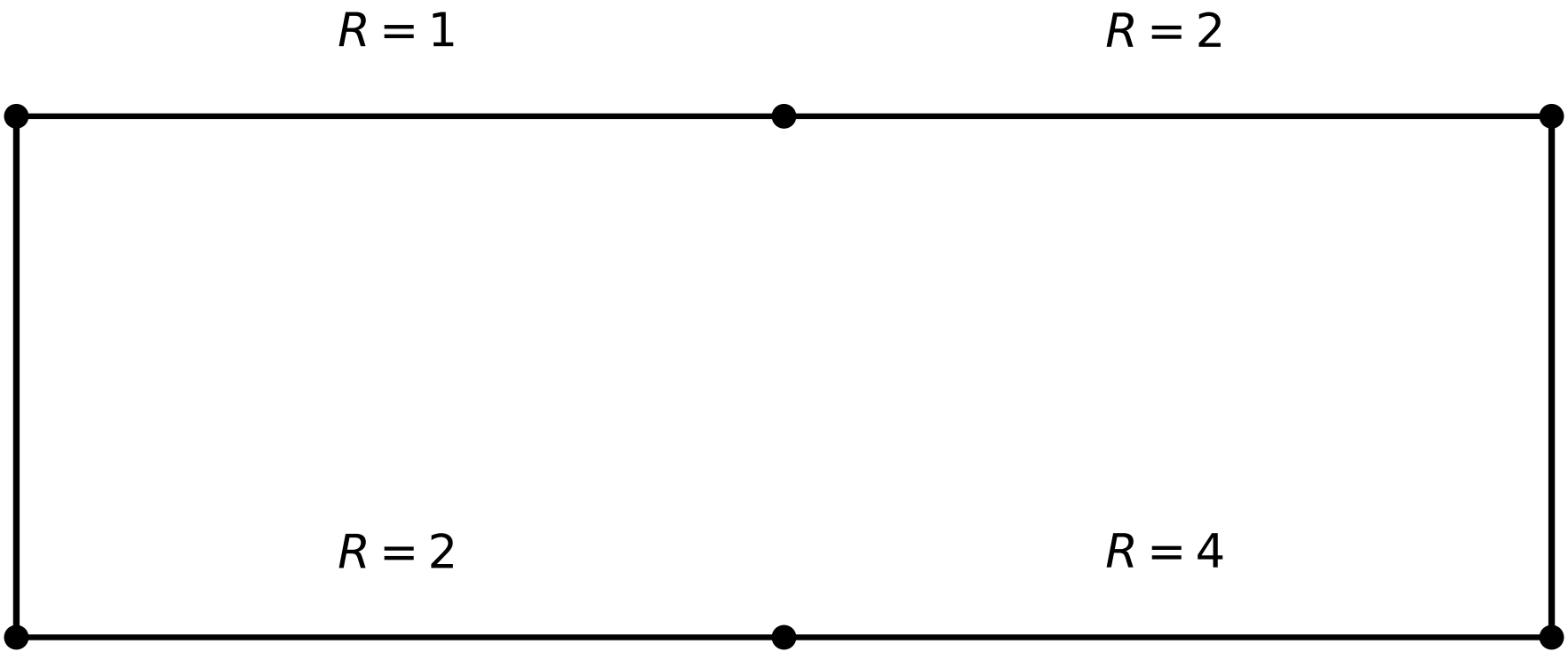

Null-support curves: length $1/n^2$
0.25
0.20
0.15
0.10
0.05
0.00
y
0.0
0.1
0.2
0.3
0.4
0.5
$x = 1/n$


Radial annulus: exponent dependence
Minimum p-energy
8
7
6
5
4
3
2
1.5
2.0
2.5
3.0
3.5
4.0
Exponent p

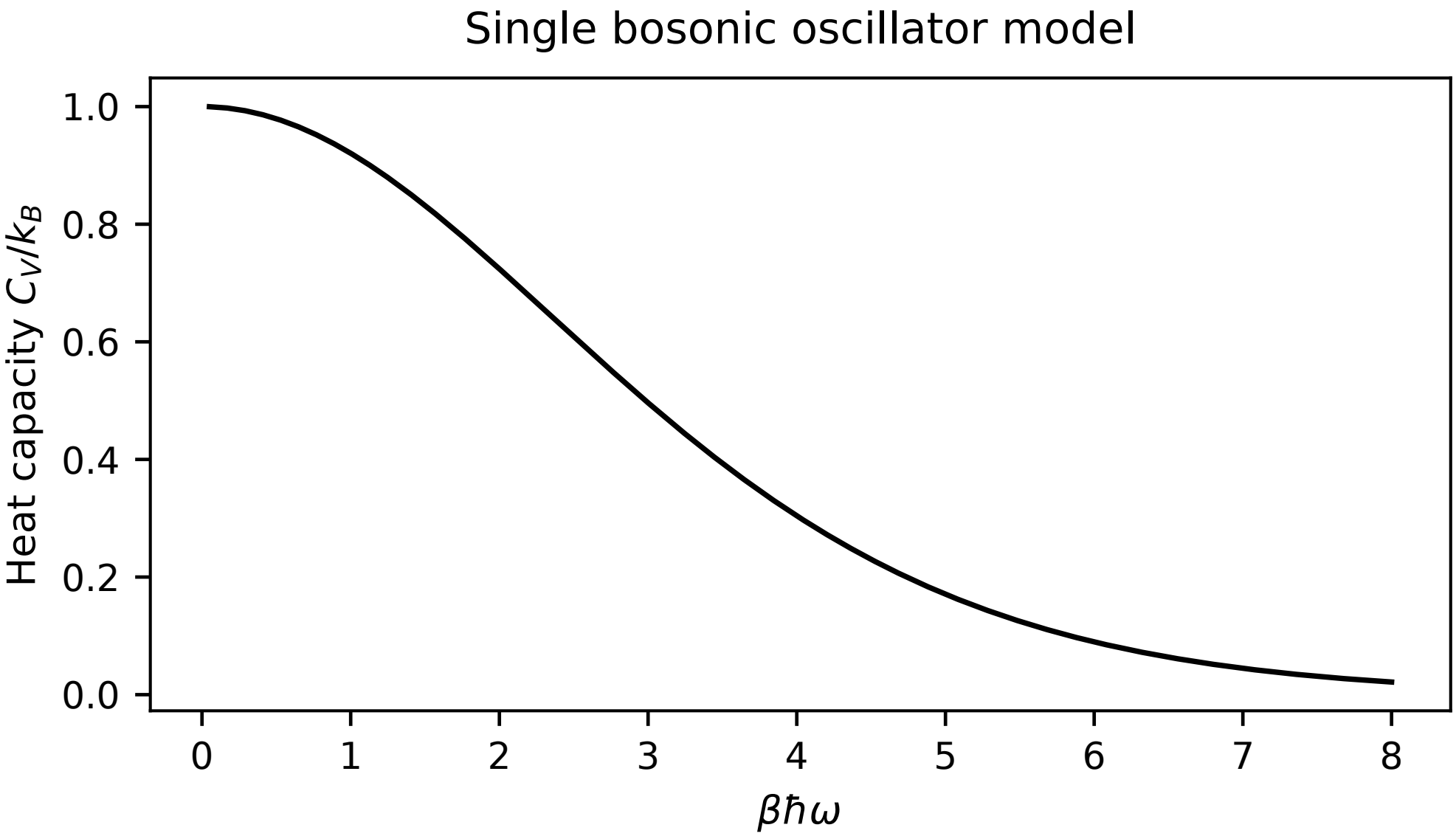

Sharp perturbation bound
Attained affine bound
Norm-resolvent bound
Relative energy-norm error
10^1
10^0
10^{-1}
0
0.0
0.2
0.4
0.6
0.8
Relative form perturbation ε

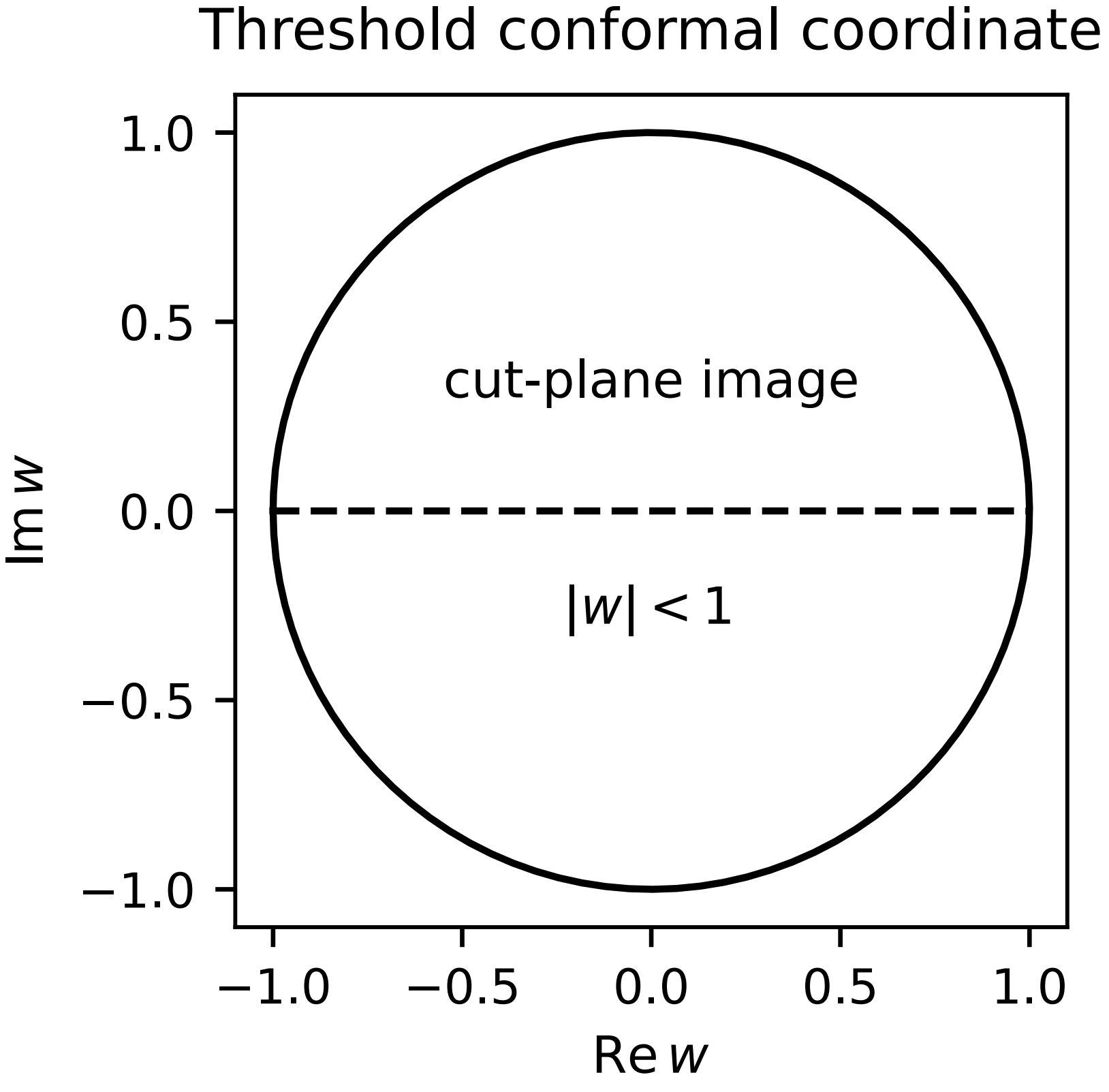

Pinching: explicit nonsharp bounds
Modulus bounds
10^{1}
10^{-2}
10^{-5}
10^{-8}
10^{-11}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
Separation / smaller diameter δ
proved lower bound
proved upper bound


Both certificates meet relative width 0.002
Method-specific work units
10^3
10^2
10^1
AGM steps per integral
Quadrature cells per integral
2
4
10
11/10
Certificate cases (categorical R)

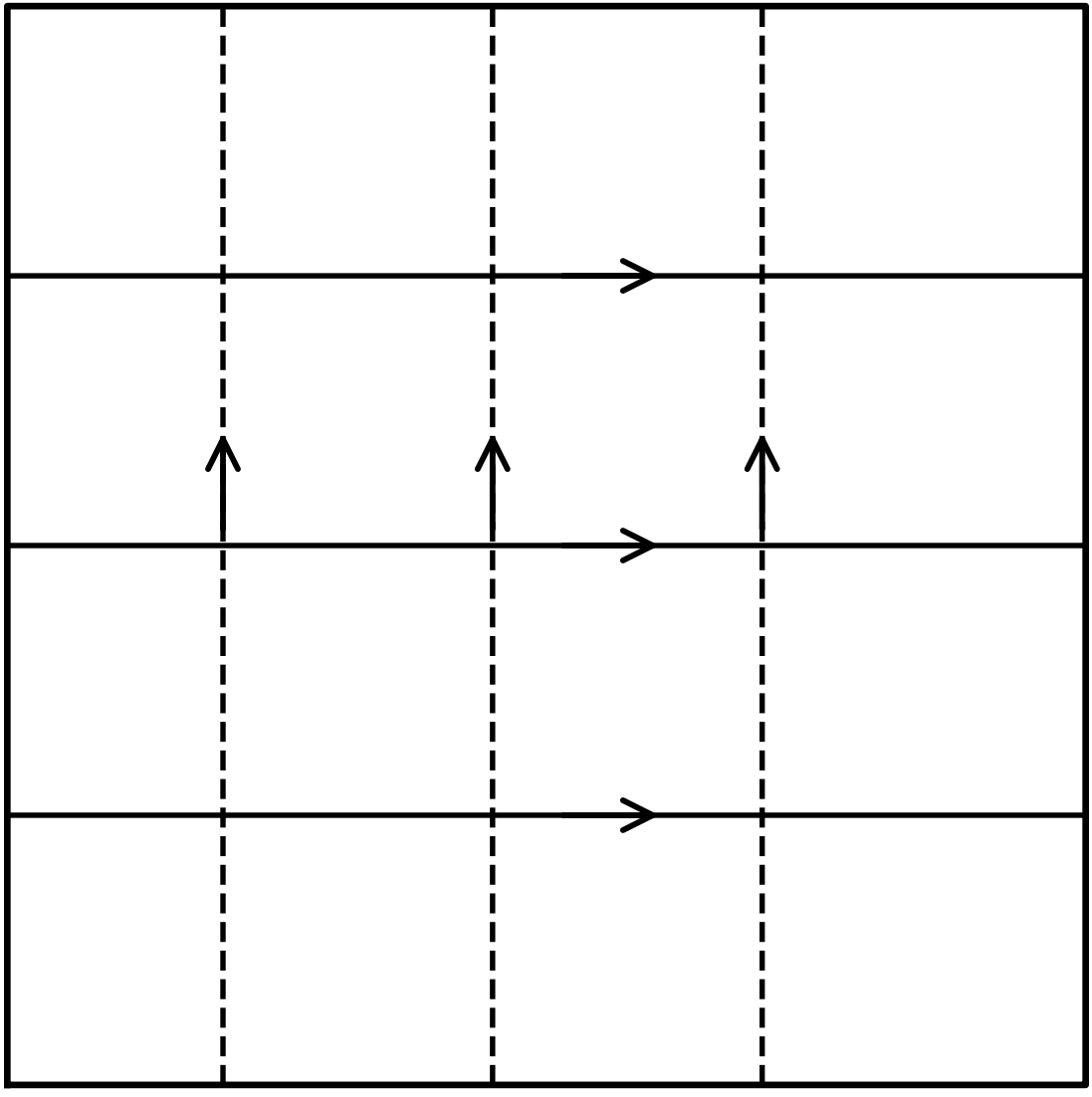


$R = 2$; unit-voltage energy $= 1/2$

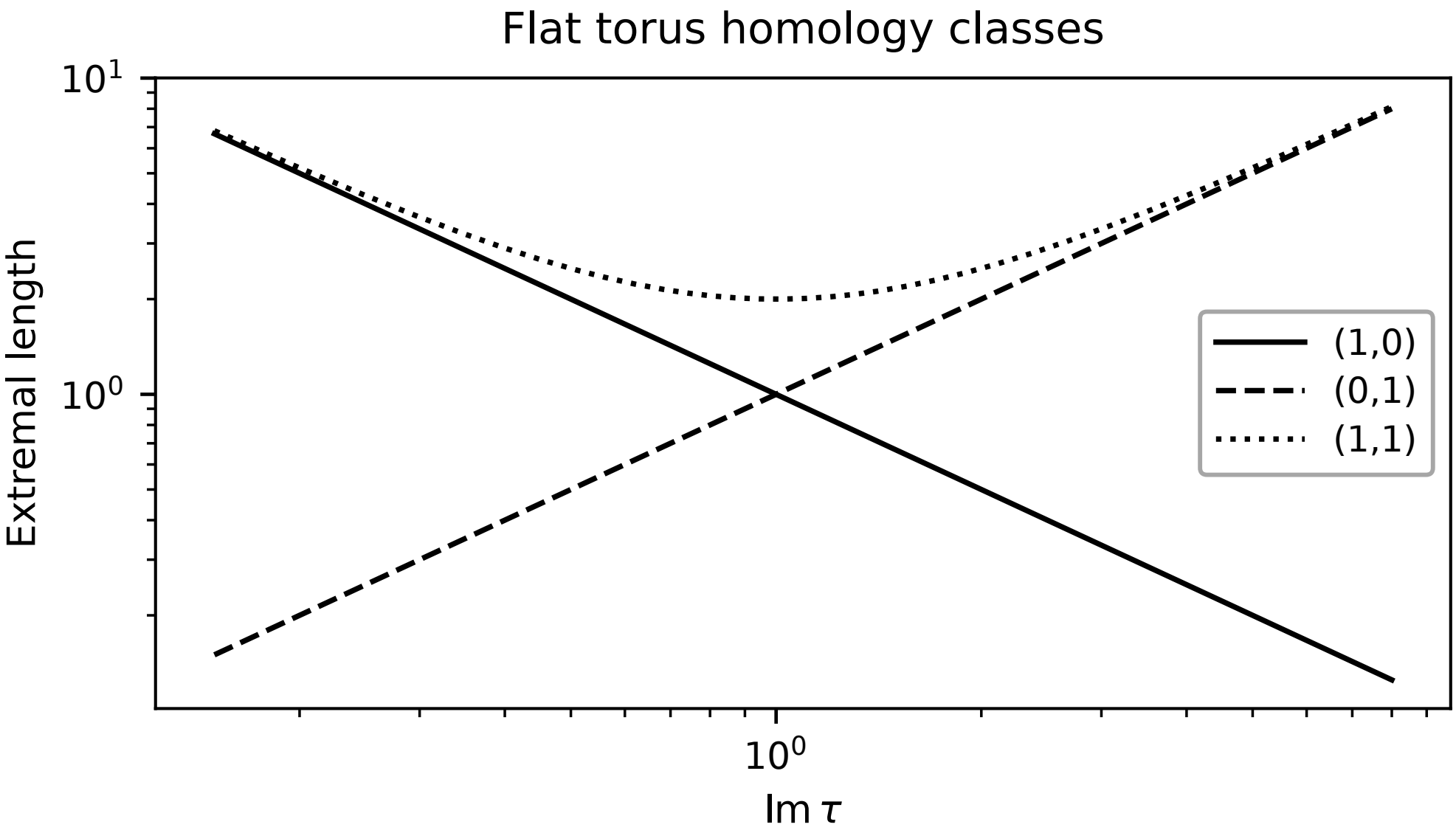

Single bosonic oscillator model
Heat capacity C_V/k_B
1.0
0.8
0.6
0.4
0.2
0.0
0
1
2
3
4
5
6
7
8
$\beta\hbar\omega$

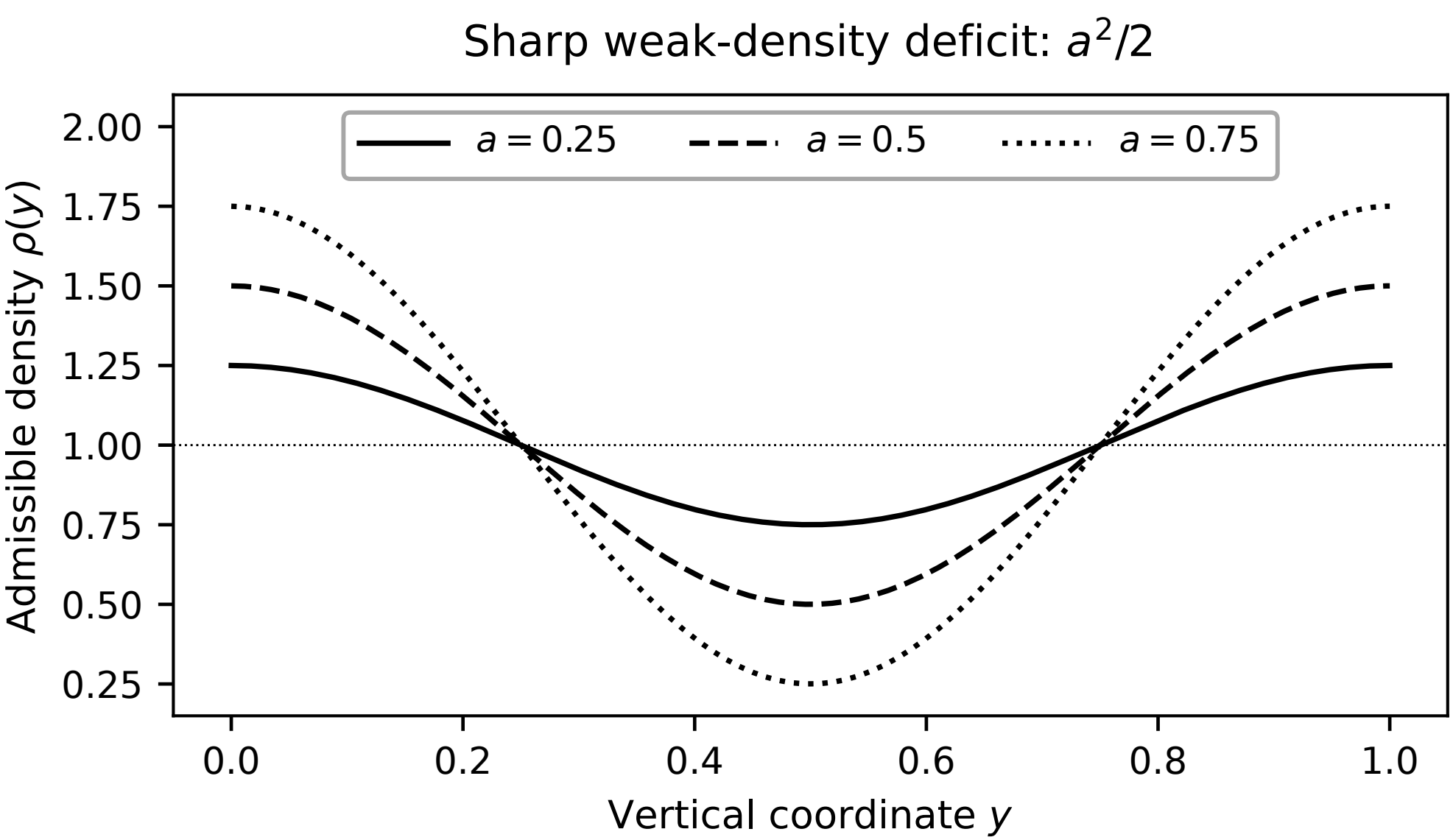

Threshold conformal coordinate
cut-plane image
$|w| < 1$
1.0
0.5
0.0
−0.5
−1.0
−1.0
−0.5
0.0
0.5
1.0
Im w
Re w

Flat torus: $W(\Gamma)W(\Delta) = 1$



Opposite sides are identified


Flat torus homology classes
10^1
10^0
Extremal length
10^0
Im τ
(1,0)
(0,1)
(1,1)


Sharp weak-density deficit: $a^2/2$
$a = 0.25$
$a = 0.5$
$a = 0.75$
Admissible density $\rho(y)$
Vertical coordinate y
2.00
1.75
1.50
1.25
1.00
0.75
0.50
0.25
0.0
0.2
0.4
0.6
0.8
1.0